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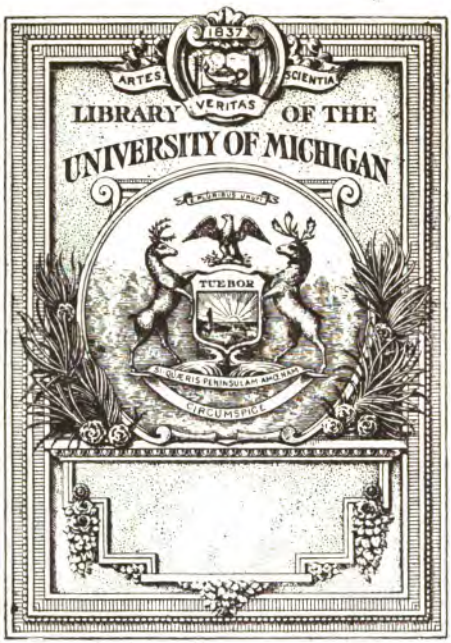
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THE

TUTOR'S GUIDE:

BEING A

COMPLETE SYSTEM OF ARITHMETIC:

WITH

VARIOUS BRANCHES IN THE MATHEMATICS.

IN SIX PARTS, viz.

I. Arithmetic in all its useful Rules, and to each a great Variety of Questions.

II. Vulgar Fractions, in all their Parts.

III. Decimal Fractions, with the Extraction of Roots of different Powers; to which are added, Rules, &c. for the easy Calculation of Interest and Annuities, &c.

IV. Mensuration of Superficies and Solids, applied to measuring Artificers' Works, &c. with a Collection of Questions for Exercise. To which is added, the Specific Gravity of Metals, &c.

V. Chronology, or the Method of finding the several Cycles, Epacts, Moveable Feasts, Time of High Water, &c.; with a Collection of Questions relating to History: likewise all the most useful Examples on both the Globes.

VI. Algebra, wherein the Method of raising and resolving Equations is rendered easy, and illustrated with a Variety of Examples and Numerical Questions.

TO WHICH IS ADDED,

AN APPENDIX,

CONTAINING DIFFERENT FORMS OF ACQUITTANCES, BILLS OF EXCHANGE, &c. &c.

The whole being designed for the Use of Schools, as a QUESTION BOOK, or a REMEMBRANCER and INSTRUCTOR to such as have some Knowledge of Figures: and adapted for the Use of the Gentleman and Scholar, as well as of the Man of Business: and recommended by several eminent Mathematicians and School-masters.

BY CHARLES VYSE,

AUTHOR OF "THE YOUNG LADIES' AND GENTLEMEN'S NEW GUIDE TO THE ENGLISH TONGUE."

THE FIFTEENTH EDITION;

CAREFULLY REVISED AND CORRECTED.

LONDON:

PRINTED FOR G. WILKIE; SCATCHERD AND LETTERMAN; LACKINGTON, ALLEN, AND CO.; J. RICHARDSON; J. M. RICHARDSON; LONGMAN, HURST, REES, ORME, AND BROWN; LAW AND WHITTAKER; SHERWOOD, NEELY, AND JONES; G. AND S. ROBINSON; GALE AND FENNER; WALKER AND CO.; AND SIMPKIN AND MARSHALL.

1815.

The MONTHLY REVIEW, *for January* 1771, *gives the following Account of the First Edition:—*

"The best method of conveying instruction is derived from experience; and though the author of the TUTOR'S GUIDE does not pretend to boast of new discoveries, yet it must be allowed that he has selected a great variety of necessary and useful rules for the obtaining a thorough knowledge in those sciences which depend upon arithmetic: and his book will be found particularly useful in this respect, as it contains a very considerable number of questions to exemplify the rules he has laid down, and to exercise the attention of the learner.—Many of them may be thought to surpass the capacity of young scholars: but this circumstance is no just objection against the book itself; it rather recommends the work to an after-review, when the understanding is enlarged and ripened. The plan and execution of Mr. Vyse's performance do honour to his judgment and application, and entitle it to the general notice of those who are intrusted with the education of youth."

The CRITICAL REVIEW *gives the following Account:—*

"Notwithstanding there are many books already extant upon the same subject, yet we apprehend that the work before us will not be deemed either unnecessary or impertinent, after having assured our readers it is recommended to the favour of the Public by one of the most considerable mathematical writers of the present age."

Printed by S. Hamilton, Weybridge, Surrey.

PREFACE.

WHEN we consider the utility of ARITHMETIC, on which science almost all the others do absolutely depend, we need not be surprised that so many efforts have been made to bring this useful branch of learning to the utmost degree of perfection: and although the vast extent of the subject does in some measure defeat these attempts, yet, upon account of its real value and use, it certainly merits all the study and pains that can be bestowed upon it.

In the following pages I have delivered the Definitions and Rules in as brief and concise a manner as I possibly could, so as to make them general; and they follow in the same order as specified in the Table of Contents. Thus, Book the First contains the four primary rules, *i.e.* Addition, Subtraction, Multiplication, and Di-

vision, in Integers; and Reduction, ascending and descending, with the Tables of Money, Weights, Measures, &c. with which the pupil should be well acquainted, before he proceeds to the use of those rules in Compound Numbers.

In Book the Second, the rules follow in the same order in which they are generally taught in schools; but I apprehend the most expeditious method for the pupil would be to learn as far as the Rule of Three, then Vulgar and Decimal Fractions, and the Extraction of the Square and Cube Roots; after which a larger field may be opened to him, as he will be more capable to judge for himself, and, with a little assistance from the master, go through any rule, to his own satisfaction, and his teacher's honour. But in schools the master very seldom either knows the business for which his scholars are designed, or the length of time they are to continue at school, and is therefore obliged to pursue the old beaten path, and teach them first what is not perhaps the most essential.

In this work, amongst several hundred questions, are all those most excellent ones of the late MARTIN CLARE, arranged according to the

several rules to which they appertain; a thing wished for by school-masters and teachers in general.

I pretend not to boast of new discoveries, but flatter myself to have selected every necessary and useful rule or proposition for obtaining a thorough knowledge in those sciences which depend upon Arithmetic; and have given a great variety of such questions as will enable the tutor to supply his pupils with those that may be most conducive to the station of life for which they may be designed.

I have selected the most difficult questions of each rule, as exercises for leisure hours.

And, in order to make this book as useful as possible, I have added several examples of the different Forms of Acquittances, Promissory Notes, Bills of Exchange, Bills of Parcels, &c. the frequent copying of which I would recommend to the pupil.

I have not adjoined the answers to the questions, as I have published (by desire of several mathematicians and school-masters) a KEY to the TUTOR'S GUIDE, wherein all the answers are worked at length, with proper references to the questions as they stand in this edition.

I need not point out the great utility of such a performance; it is sufficiently evident to all who are employed in this branch of education, especially such as have the care of a numerous school, and experience the difficulties that teachers in general labour under with respect to time; a grievance that must continue, while their remuneration is so very inadequate to their labours.

I hope the gentlemen of the profession will do me the justice to believe, that my design in this performance was not to dictate to, but ease the master; and the skilful teacher will no doubt vary the work of the questions according to the nature of the several rules.

The TUTOR'S GUIDE and its KEY furnish a Complete System of ARITHMETIC, and will enable those, who are acquainted with the first principles, to attain (without the assistance of a master) a competent knowledge of the several Rules with ease and precision.

The favourable reception the former editions have met with, encourages me to hope that the alterations and corrections which have been made in this, will merit the approbation of those gentlemen, who have kindly interested them-

selves in behalf of my former endeavours, to whom I return my most sincere thanks; and remain,

With the utmost Esteem and Respect,

Their and the Public's much obliged,

Obedient humble Servant,

CHARLES VYSE.

London, 1807.

Charles Vyse

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EXPLANATION *of the* CHARACTERS *made use of in this* BOOK.

Signs.	Names.	Significations.
$+$	*Plus*, or *more*,	The sign of Addition, as $6+2$ is 8.
$-$	*Minus*, or *less*,	The sign of Subtraction, as $6-2$ is 4.
$\times$	*Multiplied into* or *by*	The sign of Multiplication, as 6×2 is 12.
$\div$	*Divided by*,	The sign of Division, as $6\div 2$ is 3.
$=$	*Equal to*,	The sign of Equality, as $6+2=8$.
$:$ $::$	*Is to*, *So is*,	The Signs of Proportionals, as $2 : 4 :: 6 : 12$.
$\surd$	*Extraction of the Roots*,	The Square Root of $9=3$ is $\sqrt[2]{9}=3$, and the Cube Root of $8=2$ is $\sqrt[3]{8}=2$.

$\overline{6-4\times 9}=18$; signifies, that 6, less 4, multiplied by $9=18$.

N. B. 8^2 denotes that 8 is to be squared.

9^3 denotes that 9 is to be cubed.

$\sqrt[3]{4}$, or $4^{\frac{1}{3}}$, denotes the Cube Root of the Number 4.

The Twelve SIGNS *of the* ZODIAC.

♈ Aries, the Ram.
♉ Taurus, the Bull.
♊ Gemini, the Twins.
♋ Cancer, the Crab.
♌ Leo, the Lion.
♍ Virgo, the Virgin.
♎ Libra, the Balance.
♏ Scorpio, the Scorpion.
♐ Sagittarius, the Archer.
♑ Capricornus, the Goat.
♒ Aquarius, the Water-bearer.
♓ Pisces, the Fishes.

THE

TUTOR'S GUIDE;

BEING

A COMPLETE SYSTEM OF ARITHMETIC.

BOOK I.

Arithmetic in Whole and Compound Numbers.

INTRODUCTION.

ARITHMETIC is the science or knowledge of Numbers, which is either Unit, or a multitude of Units.

Unit is any thing considered as one, or 1.

Digits, or Figures, are the marks by which Numbers are denoted or expressed, and are the nine following, viz. 1, 2, 3, 4, 5, 6, 7, 8, 9: with these there is used the mark 0, called a Cipher, which of itself stands for nothing; but being annexed to the right-hand of a Digit, alters its value; thus, 40 signifies forty, and 400 stands for four hundred, &c. (See the following Table.)

Integers, or whole Numbers, are such as express a Number or multitude of things, whereof each is considered as an Unit. Thus, 6 Pounds, 12 Yards, 140 Miles, &c. each of which is called an Integer, or whole Number.

Compound Numbers are such as consist of different denominations, as Pounds, Shillings, Pence, and Farthings; or Hundreds, Quarters, Pounds, Ounces, &c.

Thus 47l. 12s. 6¾d. or 4C. 2qrs. 14lb. &c.

A Fraction, or broken Number, is always less than Unit, as ¾ represents three quarters of any thing or Unit, and $\frac{6}{8}$ is six eighths of Unit, or 1, &c.

Arithmetic, with regard to art and science, consists both in Theory and Practice.

Theory considers the nature and quality of Numbers, and demonstrates the reason of practical operations.

The Practice is that which shows the method of working by Numbers, so as to be the most useful and expeditious for

business, and has five principal or fundamental Rules for the operation; viz.

1. NUMERATION, or NOTATION. 2. ADDITION. 3. SUBTRACTION. 4. MULTIPLICATION. 5. DIVISION.

SECT. I. NUMERATION

TEACHETH to read or express the true value of any Number when written down; and consequently to write down any proposed Number, according to its true value; and this consisteth of two parts:

1. The due order of placing down Figures.
2. The true value of each Figure in its place; both of which are plainly exhibited in the following

TABLE.

Hundreds of Millions.	Tens of Millions.	Millions.	.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	.	Hundreds.	Tens.	Units.
9	8	7	.	6	5	4	.	3	2	1
4	0	0	.	0	0	0	.	0	0	0
	4	0	.	0	0	0	.	0	0	0
		4	.	0	0	0	.	0	0	0
				4	0	0	.	0	0	0
					4	0	.	0	0	0
						4	.	0	0	0
								4	0	0
									4	0
										4

EXAMPLES.

Write down the Value of the following Numbers in Words at length, viz.

(1) 94
(2) 702
(3) 3024
(4) 37460
(5) 142613
(6) 6040390
(7) 47639121
(8) 790401950
(9) 79041955

In Figures, express

(1) Seventy-seven. (2) Four hundred and ninety. (3) Six thousand and fifty-five. (4) Seventeen thousand seven hundred and nine. (5) Eight hundred thousand and two. (6) Seven millions forty-four thousand and seventy-four. (7) Six hundred and ninety-four millions four hundred thousand and sixty.

NOTATION

by Roman *Numerical Letters.*

One, five, ten, fifty, hundred, five hundred, thousand.

I, V, X, L, C, D, M.

When a lesser numerical letter stands before a greater, it must be taken from it; as, I before V or X; and X before L, or C, &c. Thus,

four, nine, forty, ninety, &c.

IV, IX, XL, XC.

When a lesser numerical letter stands after a greater, it is to be added to it. Thus,

six, eleven, sixty, one hundred and ten.

VI, XI, LX, CX.

A Line drawn over any number less than a thousand signifies so many thousands; as $\overline{LX}$, is sixty thousand; $\overline{C}$, is one hundred thousand; $\overline{M}$, is one Million, &c.

Write down in common Figures the following Numbers, expressed in numerical letters, viz. XIX, CC, DC, DLX. MI, MDCCL, $\overline{LXX}$, $\overline{CX}$, $\overline{MD}$, and $\overline{MDC}$.

Write down in numerical letters the following Numbers, expressed in common Figures, viz.

29, 104, 419, 1741, 2007, 17678, 10004, and 674084.

II. ADDITION of INTEGERS

TEACHETH to add sundry Numbers together into one Sum, called the Total.

RULE.

1. Place all the Numbers of a like name under one another; that is, units under units, tens under tens, hundreds under hundreds, &c.

2. Begin with the Units, and singly collect the Sum of each

row; and, if their Sum be less than ten, set it down underneath its own Place; but, if it exceed ten, the excess is only to be set down, carrying one for every ten to the next row, and so on, continuing to the last row, at which set down the total Amount.

PROOF.

Vary the adding, by beginning at the top of the Sum, and reckon the Figures downwards, in the same manner as you added them upwards; and if the Sum come the same as before, it is supposed to be right.

TABLE of ADDITION,

which is to be got by heart by those who are beginners in this science.

0	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	—	4	5	6	7	8	9	10	11
3	—	—	6	7	8	9	10	11	12
4	—	—	—	8	9	10	11	12	13
5	—	—	—	—	10	11	12	13	14
6	—	—	—	—	—	12	13	14	15
7	—	—	—	—	—	—	14	15	16
8	—	—	—	—	—	—	—	16	17
9	—	—	—	—	—	—	—	—	18

The manner of using the Table is thus:—Take the greater of the two Digits, whose Sum is sought, in the upper line, and the lesser on the left-hand column, in the same line with this; and underneath the other stands the Sum.

As suppose I wanted the Sum of 9 and 7: then I look for 9 on the head of the Table, and in the same line with 7, on the left-hand side, stands 16—the Sum.

EXAMPLES.

(1)	147279	(2)	176042	(3)	127492
	274042		47976		274614
	716914		274		27406
	472196		4		274
	417417		471472		24
	194746		469		4158

EXAMPLES.

(4)	147747	(5)	2147426	(6)	174684
	74774		2749		147129
	2467		275		2984
	915		3746		100
	20		74		63
	6		2147		1074
(7)	174264	(8)	14768412	(9)	174684
	7416		2131596		26276
	271		294168		174168
	147419		274		276
	4176		71471041		7
	47		219816		741705
	7913		1427		27417
	274		70		3570

III. SUBTRACTION

TEACHETH to take a lesser Number from a greater, and thereby shows the Difference or Remainder.

RULE.

1. Place your Numbers according to the direction given in Addition.

2. Begin at the right-hand, and subtract each under Figure from that which stands over it, writing each Remainder under the Figure it proceeds from; so shall all the Remainder together express the Difference required.

3. But when the under Figure exceeds that which stands over it, you must borrow ten (the same which you stopped at in Addition), from which take the lower Figure, and to that Difference add the upper Figure and set down the Sum, always remembering to carry or add one to the next Figure on the left-hand before you subtract.

PROOF.

To the lesser Number add the Remainder; if the Sum be like the greater, the work is right.

TABLE of SUBTRACTION.

	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8
1	-	0	1	2	3	4	5	6	7
2	—		0	1	2	3	4	5	6
3	—	—		0	1	2	3	4	5
4	—	—	—		0	1	2	3	4
5	—	—	—	—		0	1	2	3
6	—	—	—	—	—		0	1	2
7	—	—	—	—	—	—		0	1
8	—	—	—	—	—	—	—		0
9	—	—	—	—	—	—	—	—	

The manner of using this Table is the same with that of Addition; only, instead of adding the Digits together, subtract them.

EXAMPLES.

(1) From 1472742
Take 1251620

Rem. 221122

Proof 1472742

(2) 1704941
807467

(3) 17406542
16716746

(4) Bought 10768473
Sold 7607485

Rem. unsold

(5) 2074176
1760184

(6) 7417065
4708095

(7) From 10746142
Take 1786076

Rem.

(8) 12468409
9147608

(9) 2170684
1100787

(10) From 106742740
Take 74760946

(11) 214200040
107400760

IV. MULTIPLICATION

TEACHETH how to increase any one Number by another, so often as there are Units in that Number by which the one is increased; and serves instead of many Additions.

To this Rule belong three principal Numbers, viz.

1. The Multiplicand, or Number to be increased or multiplied.

2. The Multiplier, or Number by which the Multiplicand is increased or multiplied.

3. The Product, or Number produced in multiplying.

Note.—Before any operation can be performed in this Rule, it is absolutely necessary that the following Table be got by heart; as the ready performance of this and all the following Rules entirely depends upon having a perfect knowledge of it.

TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3		9	12	15	18	21	24	27	30	33	36
4		—	16	20	24	28	32	36	40	44	48
5		—	—	25	30	35	40	45	50	55	60
6		—	—	—	36	42	48	54	60	66	72
7		—	—	—	—	49	56	63	70	77	84
8		—	—	—	—	—	64	72	80	88	96
9		—	—	—	—	—	—	81	90	99	108
10		—	—	—	—	—	—	—	100	110	120
11		—	—	—	—	—	—	—	—	121	132
12		—	—	—	—	—	—	—	—	—	144

USE of the TABLE.

Seek the greater of the two Digits in the upper line, and underneath it, against the lesser, taken in the left-hand column, is the Product sought. Thus, to multiply 9 by 6, seek 9 in the upper line, and under it, against 6 on the left, is 54 the Product: and so of any other.

Note.—For the conveniency of dividing by 11 or 12, I have continued the Table to 12 Times, or else in Multiplication it is only required to 9 Times.

Case 1. To multiply by a single Figure.

RULE.

1. Place the Multiplier underneath the Unit's Place of the Multiplicand.

2. Multiply the Unit Figure of the Multiplicand by the Multiplier; if their Product be less than ten, set it down un-

der its own Place of Units; but if their Product exceed ten or tens, then set down the Excess only (as in Addition), and bear or carry the said ten or tens in mind, until you have multiplied the next Figure of the Multiplicand by the same Figure of the Multiplier, and to their Product add one for each ten borne in mind, setting down the excess of their Sum above ten, or tens, as before; and so proceed until all the Figures of the Multiplicand are multiplied by the Multiplier.

PROOF.

The most sure and unerring way is by Division. But as the learner is supposed not yet to know that Rule, he cannot prove by it; let him therefore make the Multiplicand the Multiplier, and if the Product come out the same as before, the Work is right.

Some Masters that teach, and several Authors that write of, Arithmetic, prove Multiplication by the Cross. But this method is not to be depended upon, as it will prove a Sum to be right, when at the same time the work is utterly false. But it will not prove a Sum false that is right.

EXAMPLES.

(1) Multiplicand 417609853		(2) 342719086
Multiplier 2		3
Product		
(3) 257680914	(4) 174205683	(5) 170965381
4	5	6
(6) 749185603	(7) 262758094	(8) 376290845
7	8	9

Case 2. When the Multiplier consists of several Figures.

RULE.

1. Place each Figure in the Multiplier respectively under its own kind in the Multiplicand.

2. Multiply the Multiplicand by each Figure of the Multiplier, as before; observing to place the first Figure of each

respective Product underneath that Figure of the Multiplier by which you multiply.

3. Add the several Products together, and the Sum will be the desired, or total, Product.

EXAMPLES.

(9)	142737396	(10)	21607472	(11)	12076849
	16		28		97
(12)	247567	(13)	317649	(14)	2706915
	475		689		3746
(15)	147678	(16)	47269	(17)	73581
	5682		73581		47269
(18)	764258	(19)	417396	(20)	2719064
	417396		764258		5648736

Case 3. When Ciphers are intermixed with the Figures in the Multiplier.

RULE.

Omit them, and place the first Figure of each particular Product under its respective Multiplier.

EXAMPLES.

(21)	10746047	(22)	804700625
	40500108		207008009

Case 4. When there are Ciphers at the right-hand of either, or both the Multiplier and the Multiplicand.

RULE.

Proceed as before, neglecting the Ciphers until the particular Products are added together, and to that Sum place the Number of Ciphers that are at the end of both Factors, on the right-hand.

EXAMPLES.

(23) 1460900
8700

(24) 2768000
24600

If it be required to multiply any Number by 10, 100, 1000, &c. it is only annexing the Ciphers of the Multiplier to the right-hand of the Multiplicand, and the work is done.

Case 5. When the Multiplier is such a Number that any two Figures in the Table, being multiplied together, will produce it.

RULE.

Multiply the given Number by one of those Figures, and that Product by the other; which will give the desired Product.

EXAMPLES.

(25) Multiply 24674 by 16.

(26) Mul. 340764 by 28.

(27) Mul. 142395 by 56.

(28) Mul. 176848 by 81.

(29) Mul. 420746 by 72.

(30) Mul. 17093 by 63.

(31) Mul. 43074 by 144.

(32) Mul. 14068 by 132.

Case 6. When the Multiplier is any Number between 10 and 20.

RULE.

Multiply by the Figure in the Unit's Place, and, as you multiply, add to the Product of each single Figure that of the Multiplicand, which stands next on the right-hand.

EXAMPLES.

(33) 142716
11

(34) 14276
12

(35) 146094
13

(36) 24176
14

(37) 36142
15

(38) 176424
16

(39) 14609
17

(40) 18627
18

V. DIVISION

TEACHETH us to find how often one Number is contained in another, or to divide any Number or Quantity given into any Parts assigned, and serves instead of many Subtractions. In this Rule there are three Numbers real, and a fourth accidental; viz.

1. The Dividend, or Number to be divided.
2. The Divisor, or Number by which you divide.
3. The Quotient, or Number that shows how often the Divisor is contained in the Dividend.
4. The Remainder, which is always less than what you divide by.

Case 1. When the Divisor is not greater than 12.

RULE.

First seek how often the Divisor is contained in the first Figure of the Dividend, or, in case the first Figure of the Dividend be less than the Divisor, then in the first two Figures of the Dividend, and set the quotient Figure down accordingly; and, if any thing remain, carry it to the next Figure in the Dividend, where it must be reckoned as so many Tens; that is, if one remain, you call it 10; if two, 20; if five, 50, and so on; bearing in mind the Remainder of each Figure, and adding it to the next, until you have made use of all the Figures in the Dividend. This is called Short Division.

PROOF.

Multiply the Quotient by the Divisor, and, as you multiply, add the Remainder, if any, or add the whole Remainder to the Product at last, and if it come the same as the Dividend, the Work is right.

EXAMPLES.

(1)	2)1742636	(2)	3)2764064	(3)	4)2160742
(4)	5)1076426	(5)	6)71420954	(6)	7)4674263

(7) 8)2768096 (8) 9)6768094 (9) 10)2762764

(10) 12)276484

Case 2. When the Divisor consists of many Places or Figures.

RULE.

1. If the Divisor be a less Number than so many Figures taken in the Dividend, see how often the first Figure of the Divisor is contained in the first Figure of the Dividend, and the Figure which expresses it is the first of the Quotient; by which multiply the Divisor, and place the Product under the said Figures of the Dividend, and draw a line underneath it: subtract it therefrom, and to the remainder annex the following Figure of the Dividend, then proceed as before.

2. But if it happen that the Divisor be a greater Number than so many Figures of the Dividend; you must take a Number of places in the Dividend greater by one, and see how often the first Figure in the Divisor is contained in the first two of the Dividend, allowance being made for what you carry from the Figure on the right.

3. If in any Case the Remainder be so small, that when the Figure of the Dividend, joined with it, makes a Sum less than the Divisor, then a Cipher is to be placed in the Quotient, and another Figure brought down, and then proceed as before; this is called Long Division.

EXAMPLES.

(11) 25)736473575((16) 7489)1204530760(
(12) 84)35730972((17) 42163)112737328(
(13) 648)272357640((18) 61745)392628787(
(14) 759)30891829676((19) 684573)3233238699(
(15) 3065)63463902247((20) 476085)98839054780(
(21) 4728395)27750950255(

Case 3. When the Divisor has Ciphers on the right-hand.

RULE.

Strike them off, and so many of the last Figures in the Dividend: divide by those Figures of the Divisor that are left when the Ciphers are omitted. But when the Division is

ended, those Ciphers so omitted in the Divisor, and the Figures cut off in the Dividend, are both to be restored to their own places.

EXAMPLES.

(22) 2800)11928248((23) 172000)247004674(

Note—When the Dividend has the same Number of 0's on the right-hand as the Divisor, strike them off from each, and the Remainder will be so many of what you divide by, without annexing the 0's that were struck off.

EXAMPLES.

(24) 473000)351858000((25) 6970000)599430000(

Case 4. When the Divisor is such a Number, that any two Figures in the Multiplication Table, being multiplied together, will produce the said Divisor.

RULE.

Divide the given Number by one of those Figures, and that Quotient again by the other, which will give the Quotient required.

Note.—Observe, that if there be a Remainder in the last Division, it will be so many times the first Divisor; which, added to the first Remainder, if any, will give the true one.

EXAMPLES.

(26)	Divide	1206816	by 16	(30)	Divide	247684	by 28
(27)		42768	48	(31)		14652	64
(28)		74682	72	(32)		417681	81
(29)		14276	144	(33)		307684	132

Case 5. When the learner is pretty well versed in Division, he may subtract each Figure of the Product, as he produces it, and so only write the Remainder, which will shorten the Work, and be much the best way, when the Divisor is small.

EXAMPLES.

(34) 17)690489((36) 467)2148686(

(35) 86)5343698((37) 6074)24939844(

TABLES of ENGLISH COINS, &c.

Marked			
q.	4 Farthings	are one	Penny.
d.	12 Pence		Shilling.
s.	20 Shillings		Pound, £.

$\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ is wrote for 1q., 2q., 3q.

PENCE TABLE.

d.		s.	d.	d.		s.
20	are	1	8	24	are	2
30		2	6	36		3
40		3	4	48		4
50		4	2	60		5
60		5	0	72		6
70		5	10	84		7
80		6	8	96		8
90		7	6	108		9
100		8	4	120		10
110		9	2	132		11
120		10	0	144		12

The WEIGHT and VALUE of such GOLD and SILVER COINS as are most commonly used in ENGLAND.

	Weight.			Value.		
	dwts.	*gr.*	*mites.*	*l.*	*s.*	*d.*
A Guinea - - -	5	9	9 - - - -	1	1	0
Half ditto - - -	2	16	$14\frac{1}{2}$ - - - -	0	10	6
A Quarter ditto	1	8	7 - - - - -	0	5	3

SILVER.

A Crown - - -	19	8	$10\frac{1}{2}$ - - - -	0	5	0
Half ditto - -	9	16	$5\frac{1}{4}$ - - - -	0	2	6
A Shilling - -	3	20	18 - - - -	0	1	0
A Sixpence - -	1	22	9 - - - -	0	0	6

Note.—Twenty Mites make one Grain.

We also have had Portugal Money in use here, the Value and Weight of which are as follow:

	l.	*s.*	*d.*		*dwts.*	*grs.*
A Piece of - -	3	12	0	should weigh	18	16
Ditto of - -	1	16	0	- - - - - - - -	9	8
Ditto of - -	0	18	0	- - - - - - - -	4	15
Ditto of - -	0	9	0	- - - - - - - -	2	7½
A Moidore - -	1	7	0	- - - - - - - -	6	18
Half ditto -	0	13	6	- - - - - - - -	3	9
Quarter ditto	0	6	9	- - - - - - - -	1	16½

A Pound of Copper Avoirdupois is coined into twenty-three Pence; consequently a Halfpenny is one third of an Ounce nearly, and a Farthing one sixth.

	s.	*d.*			*l.*	*s.*	*d.*
A Noble	6	8	A Carolus		1	3	0
Angel	10	0	Jacobus		1	5	0
Mark	13	4	Groat		0	0	4

VI. REDUCTION

TEACHETH to reduce all great Numbers into small, by multiplying the given Number by so many of the next lower Name as make one of the higher; still keeping them equivalent in Value, and is called Reduction descending: on the contrary, all small Names are brought into great, by dividing the given number by so many of the lesser Name as make one of the next greater; this is the reverse of the last, and is termed Reduction ascending.

EXAMPLES in MONEY.

(1) In 27*l.* how many Shillings and Pence?
(2) Reduce 6480 Pence to Shillings and Pounds.
(3) How many Shillings, Pence, and Farthings, are there in 40*l.* 10*s.*?
(4) In 38880 Farthings how many Pounds?
(5) Reduce 104*l.* 17*s.* 6¾*d.* to Farthings.
(6) How many Pounds in 100683 Farthings?
(7) In 21 Guineas, how many Shillings, Pence, and Farthings?
(8) Reduce 21168 Farthings to Guineas.
(9) In 42 Moidores how many Farthings?
(10) How many Moidores in 54432 Farthings?

WEIGHTS and MEASURES.

TROY WEIGHT.

Marked			
gr.	24 Grains	make one	Penny-Weight.
dwts.	20 Penny-Weights		Ounce.
oz.	12 Ounces		Pound, ℔.

By this Weight are weighed Gold, Silver, Jewels, Amber, and all Liquors.

N. B. 14*oz.* 11*dwts.* 15½*grs.* Troy, are equal to 1 Pound Avoirdupois.

EXAMPLES.

(1) In 24*lb.* of Silver, how many Ounces, Penny-weights, and Grains?

(2) Reduce 138240 *grs.* to *dwts. oz.* and *lb.*

(3) In an Ingot of Silver, weighing 12*lb.* 10*oz.* 22*grs.* how many Grains?

(4) Reduce 73942 *grs.* to Pounds.

APOTHECARIES WEIGHT.

Marked			
grs.	20 Grains	make one	Scruple.
℈	3 Scruples		Dram.
ʒ	8 Drams		Ounce.
℥	12 Ounces		Pound, ℔.

Apothecaries, in making up their medicines, use this Weight, but they buy and sell their drugs by the Avoirdupois Weight.

EXAMPLES.

(5) In 14*lb.* how many Ounces, Drams, Scruples, and Grains?

(6) Reduce 80640 *grs.* to ℈, ʒ, ℥, and ℔.

(7) How many Grains in 4 ℔, 11 ℥, 2 ℈, 17 *grs.*?

(8) In 28377 *grs.* how many Pounds?

AVOIRDUPOIS WEIGHT.

Marked			
dr.	16 Drams	make one	Ounce.
oz.	16 Ounces		Pound.
lb.	28 Pounds		Quarter of Cwt.
qr.	4 Quarters or 112lb.		Hundred.
cwt.	20 Hundred		Ton.

By Avoirdupois Weight is weighed all manner of things that have waste; as all Physical Drugs and Grocery, Resin, Wax, Pitch, Tar, Tallow, Soap, Hemp, Flax, Hay, Wool, &c.; all base Metals and Minerals, as Iron, Steel, Lead, Tin, Copper, Alum, Copperas, &c. Also Bread, Butter, Cheese, Salt, Butchers' Meat, &c.

The Denominations in some of which are as follow, viz.

8 Pounds	make one	Stone of Butchers' Meat.
14 Pounds		Stone of Horseman's Weight.
19½ Hundreds		Fother of Lead.

WOOL WEIGHT.

7 Pounds	make one	Clove	6½ Todds	make one	Wey.
2 Cloves		Stone	2 Weys		Sack.
2 Stones		Todd	12 Sacks		Last.

HAY.		BREAD.	Weight. *lb.*	*oz.*	*dr.*
56 Pounds of Old Hay, or 60 Pounds of New ditto	make Truss	Peck Loaf	17	6	1
		Half ditto	8	11	0½
36 Trusses make 1 Load.		Quartern ditto	4	5	8¾

Note.—There are some Sorts of Silk which are weighed by a great Pound of 24 oz.

EXAMPLES.

(9) In 1 Ton, or 20 *Cwt.* how many Quarters, Pounds, Ounces, and Drams?
(10) Reduce 573440 *drs.* to Hundreds, &c.
(11) Reduce 27 *lb.* 12 *oz.* 11 *drs.* to Drams.
(12) How many Pounds in 7115 *drs.*?
(13) In 12 Tons, 10 *Cwt.* 14 *lb.* 11 *oz.* 15 *drs.* how many *drs.*?
(14) How many Tons are there in 7171775 *drs.*?

CLOTH MEASURE.

			Marked
4 Nails	make one	Quarter of a Yard	na. qrs.
3 Quarters		Ell Flemish	Ell. Fl.
4 Quarters		Yard	Yd.
5 Quarters		Ell English	Ell Eng.
6 Quarters		Ell French	Ell Fr.

Scotch and Irish linens are bought and sold by the Yard; but Dutch linens are bought by the Ell Flemish, and sold by the Ell English.

EXAMPLES.

(15) In a Piece of Cloth containing 24 Yards, how many Quarters and Nails?
(16) Reduce 384 Nails to Yards.
(17) How many Nails are there in 72 *Ells Eng.* 4 *qrs.* 2 *na.*?
(18) Reduce 1458 Nails to Ells English.
(19) In 121 Ells Flemish how many Nails?
(20) Reduce 1452 Nails to Ells Flemish.
(21) How many Nails in 42 *Ells Fr.* 5 *qrs.*?
(22) Reduce 1028 Nails to Ells French.

LONG MEASURE.

Marked			
b. c.	3 Barley Corns	make one	Inch.
in.	12 Inches		Foot.
f.	3 Feet, or 36 Inches		Yard.
yd.	2 Yards, or 6 Feet		Fathom.
	5½ Yards, or 11 Half-yds.		Pole, Rod, or Perch.
p.	40 Poles, or 220 Yards		Furlong.
fur.	8 Furlongs, or 1760 yds.		Mile.
m.	3 Miles		League.
lea.	23⅓ Leagues, or 69⅓ Miles		Degree, Deg.

360 Degrees are the circumference of the Globe.
5 Feet are a Geometrical Pace.
16½ Feet are a Pole.

ALSO,

4 Inches	make one	Hand or Hand's Breadth.
3 Hands' Breadth		Foot.
1½ Foot		Cubit.
2 Cubits		Yard.

By this Measure, distances of places, or any thing else that has Length only, are measured.

EXAMPLES.

(23) In 176 *m.* 30 *p.* how many Poles?
(24) Reduce 56350 Poles to Miles.
(25) How many Yards, Feet, and Inches, are there in 200 Miles?
(26) In 12672000 Inches, how many Miles?
(27) Reduce 12 Leag. 1 M. 6 Fur. 29 P. 4 Yds. to Barley Corns.
(28) In 7193178 b. c. how many Leagues, &c.?

LAND MEASURE.

Marked			
y.	5½ Yards	make one	Perch, Rood, or Pole.
p.	40 Poles		Rood.
r.	4 Roods		Acre.
a.	30 Acres		Yard of Land.
	100 Acres		Hide of Land.

The best way of measuring Land is by a Chain of 4 Poles, or 66 Feet in length, which is divided into 100 equal parts, called Links.

Inches.	Barley Corns.		
7	2.76	make one	Link.
25	Links		Pole.
4	Poles or 100 Links or 22 Yards		Chain.
10	Chains		Furlong.

EXAMPLES.

(29) In 42 Acres, how many Roods and Poles?
(30) Reduce 6720 Poles to Acres.
(31) In 12 *A.* 3 *R.* 29 *P.* how many Poles?
(32) How many Acres in 2069 Perches?

WINE MEASURE.

Marked			
pts.	2 Pints	make one	Quart.
qts.	4 Quarts or 8 Pints		Gallon.
gal.	10 Gallons		Anker of B. or R.
	18 Gallons		Runlet.
	31½ Gallons		Barrel.
	42 Gallons		Tierce.
tier.	2 Tierces or 84 Gallons		Puncheon, Punch.
	63 Gallons		Hogshead.
hhds.	2 Hogsheads or 126 Gal.		Pipe or Butt.
p.	2 Pipes or 252 Gallons		Tun.

Note.—A Tun of Wine is 18 Cwt. Avoirdupois.
A Gallon is 231 solid Inches.

By Wine Measure, all Spirits, Mead, Perry, Cider, Vinegar, Oil, and Honey, &c. are measured; as also Milk, not by law, but custom only.

EXAMPLES.

(33) In 4 Ankers of Brandy, how many Gallons and Quarts?
(34) In 160 Quarts, how many Ankers?
(35) Reduce 4 Hhds. of Wine to Gallons and Pints.
(36) How many Hogsheads of Wine are there in 2016 Pints?
(37) Reduce 42 Tierces and 24 Gallons to Pints.
(38) How many Tierces in 14304 Pints?
(39) In 4 Tuns, 1 P. 1 Hhd. 42 Gal. 6 Pts. how many Pints?
(40) Reduce 9918 Pints to Tuns, &c.

WINCHESTER MEASURE.

Called also Ale and Beer Measure.

Marked				
pts.	2 Pints	make one	Quart.	
qts.	4 Quarts or 8 Pints	make one	Gallon.	
gal.	8 Gallons Ale, or 9 Gallons Beer	make one	Firkin.	
fir.	2 Firkins	make one	Kilderkin.	
kil.	2 Kilderkins, or 4 Firkins	or	Gal. 32 Ale, 36 Beer	make 1 Barrel.
bar.	1½ Barrel, or 3 Kilderkins	or	48 Ale, 54 Beer	make 1 Hogshead.
hhds.	2 Hogsheads or 3 Bar. or 108 Gal.	make 1	Butt.	
	2 Butts or 216 Gallons	make 1	Tun.	

Note—8½ Gallons is a Firkin of Beer or Ale, in all parts of England, except London.

A Gallon of Ale or Beer is 282 solid Inches.

A Firkin of Soap or Herrings is the same with that of Ale.

EXAMPLES.

(41) In 12 Barrels of Ale, how many Gallons and Quarts?
(42) Reduce 1536 Quarts of Ale to Barrels.
(43) In 42 Barrels of Beer, how many Pints?
(44) Reduce 12096 Pints of Beer to Barrels.
(45) In 6 Hhds. 27 Gal. 6 Pts. of Ale, how many Pts.?
(46) How many Hhds. of Ale in 2526 Pts.?
(47) How many Gal. and Pts. in 14 Hhds. 47 Gal. of Beer?
(48) Reduce 6424 Pints of Beer to Hogsheads.
(49) Reduce 6 Tuns, 1 Butt, 42 Gallons of Beer to Quarts.
(50) How many Tuns, &c. in 5784 Quarts of Beer?

DRY MEASURE.

Marked			
pts.	2 Pints	make one	Quart.
qts.	4 Quarts or 8 Pints		Gallon.
gal.	2 Gallons		Peck.
pks.	4 Pecks or 8 Gallons		Bushel.
bu.	4 Bushels		Coomb.
c.	2 Coombs or 8 Bushels		Quarter.
qrs.	5 Quarters		Wey.
	2 Weys or 10 Quarters		Last.

ALSO,

4 Quarters or 32 bu. / 2 Bushels } make one { Chaldron / Strike } of Corn.

A Load of Corn is 5 Bushels.

A Cart-load of ditto is 40 Bushels.

2 Quarts are one Pottle, both in Liquid and Dry Measure.

A Gallon contains 268$\frac{4}{5}$ solid Inches.

In measuring Sea Coal.

5 Pecks make one Bushel, Water Measure.

3 Bushels	make one	Sack.
9 Bushels		Vatt.
36 Bushels, or 12 Sacks		Chaldron.
21 Chaldrons		Score.

By Dry Measure, Corn, Salt, Coals, and all other Dry Goods, are measured.

The standard Bushel is 18$\frac{1}{2}$ Inches wide, and 8 Inches deep.

EXAMPLES.

(51) In 24 Quarters of Corn, how many Bushels, Pecks, Gallons, and Quarts?

(52) How many Quarters of Corn in 6144 Quarts?

(53) Reduce 36 chs. 26 bu. of Coals to Pecks.

(54) How many Chaldrons of Coals in 5288 Pecks?

(55) In 64 Lasts of Corn, how many Weys, Bu. and Pecks?

(56) How many Lasts in 20480 Pecks?

TIME.

Time of itself is nothing, but from Thought
Receives its Rise, by labouring Fancy wrought;
From Things consider'd, whilst we think on some
As present, some as past, or yet to come;
No Thought can think on Time, that's still confess'd,
But thinks on Things in Motion or at Rest.

Marked			
	60 Thirds	make one	Second.
sec.	60 Seconds		Minute.
m.	60 Minutes		Hour.
h.	24 Hours		Day.
d.	7 Days		Week.
w.	4 Weeks, or 28 Days		Month.
	52 Weeks, 1 Day, 6 Hours, or		
mo.	13 Months, 1 Day, 6 Hours, or		Year Julian.
	365 Days, 6 Hours		

365 Days, 5 Hours, 48 Minutes, 57 Seconds, 39 Thirds, are a Solar Year.

The Year is also divided into 12 unequal Calendar Months, called,

January, February, March, April, May, June, July, August, September, October, November, December.

And to know how many Days are in each Month, observe to get by heart the following Lines:

Thirty Days hath September,
April, June, *and* November;
February *hath twenty-eight alone,*
And all the rest have thirty-one;
Except Leap Year, and then's the Time,
February's *Days are twenty-nine.*

EXAMPLES.

(57) How many Hours, Minutes, and Seconds, are there in a Week, or 7 Days?
(58) In 60480 Seconds, how many Days?
(59) Reduce 6 mo. 4 d. to sec.
(60) In 14860800 sec. how many Months?
(61) How many Seconds are there in a Julian Year, or in 365 Days, 6 Hours?
(62) In 31557600 sec. how many Days?
(63) How many Thirds are there in a Solar Year, or in 365 Days, 5 Hours, 48 Minutes, 57 Seconds, and 39 Thirds?
(64) Reduce 189341625 9 Thirds to Days.

SQUARE or SUPERFICIAL MEASURE.

144 Square Inches	make one	Square Foot.
9 —— Feet		—— Yard.
30¼ —— Yards		—— Pole.
40 —— Poles		—— Rood.
4 —— Roods		Acre.
4840 —— Yards, or 640 Acres		Square Mile.

272¼ Feet is one Rod of Brick Work.
100 Square Feet is one Square of Flooring.

By this Measure are measured all Things in which Length and Breadth are only considered.

EXAMPLES.

(65) In 42 Square Yards, how many Square Inches?
(66) How many Square Yards in 54432 Square Inches?
(67) Reduce 3 sq. 42 Feet, 64 in. of Flooring to Inches.
(68) How many Squares are there in 49312 square Inches?

CUBIC or SOLID MEASURE.

1728 Solid Inches	make one	Solid Foot.
27 —— Feet		—— Yard.
40 —— Feet of round Timber, or		Ton or Load.
50 —— Feet of hewn Timber		

A Solid Yard of Earth is called a Load.

108 Solid Feet, i. e. 12 Feet long, 3 Feet broad, and 3 Feet deep, or commonly 14 Feet long, 3 Feet 1 Inch broad, and 3 Feet 1 Inch deep, is a Stack of Wood; 128 solid Feet, i. e. 8 Feet long, 4 Feet broad, and 4 deep, is a Cord of Wood.

By this Measure are measured all Things in which are considered Length, Breadth, and Depth or Thicknes

EXAMPLES.

(69) In 27 solid Yards, how many solid Inches?
(70) Reduce 1259712 solid Inches to solid Yards.
(71) How many solid Inches are there in 4 Tons 24 Feet of hewn Timber?
(72) In 387072 solid Inches, how many Tons of hewn Timber?

Of some particular WARES or GOODS.

12	make one	Dozen.
12 Dozen		Gross.
12 Gross		Great Gross.
20		Score.
5 Score		Hundred.
6 Score, or 120		Great Hundred.
1000		Thousand.

END OF BOOK I.

BOOK II.

EXERCISES in NUMERATION.

In Figures express the following Numbers, viz.

(1) One Million and a Half in South Sea Bonds.

(2) Threescore and twelve Thousand thirteen Hundred Weight of Lead.

(3) Fifteen Thousand and fourscore Million of Stivers.

(4) One Hundred and twenty Thousand two Hundred and six Millions seventy Thousand seven Hundred and seven Rials of Plate.

(5) Three Millions and thirty-three Thousand and Thirty Pieces of Eight.

(6) Four Hundred Thousand and forty Hundred Pounds, thirty-four Shillings, and fourteen Pence five Farthings.

ADDITION.

EXAMPLES of INTEGERS.

(1) Add the following Numbers, viz. 140724, 296, 42, 6740, 64167, 20, 2687, and 2654 together.

(2) Add 27460, 176, 2900, 274, 1004, 64, 596, 41, and 6104 together.

(3) Add 867, 317, 69, 1720, 276842, 49, 426074, and 60 together.

VII. COMPOUND ADDITION

TEACHETH to add sundry Sums or Numbers together having different Denominations: as in Money, Weights, Measures, &c.

RULE.

1. Place the Numbers of a like Denomination under each other, viz. Pounds under Pounds, Shillings under Shillings, Pence under Pence, Farthings under Farthings, &c.

RULE.

2. Begin to add at the lowest Denomination first, as in Integers, then divide that Sum by as many of the same Denomination as make one of the next greater, setting down the Remainder under the Row added; and carry the Quotient to the next superior or greater Denomination, whose Sum you must also find: proceed in this manner to the last, or greatest Denomination, which add as Integers.

EXAMPLES of MONEY.

	l.	*s.*	*d.*		*l.*	*s.*	*d.*		*l.*	*s.*	*d.*
(1)	4	17	11¾	(2)	14	11	6	(3)	127	11	10¾
	2	6	4		27	16	11½		41	17	6
	1	19	10¾		41	17	7		100	0	0
	3	11	6½		56	6	4¾		52	10	11¾
	8	17	9		17	11	11½		116	12	6
	1	4	0		47	6	4		24	19	11¾
	6	0	11¾			10	6		6	6	0
	5	12	6		4	0	0			10	6
	2	8	11¾		17	17	6¾		2	2	0

(4) Add 270*l.* 16*s.* 6½*d.* 60*l.* 10¾*d.* 60*l.* 10*s.* ¾*d.* 96*l* 6*s.* 10*d.* 176*l.* 6*s.* 6¾*d.* 2*l.* 2*s.* 16*l.* 17*s.* 6½d. and 100*l.* into one Sum.

(5) Add 260*l.* 17*s.* 67*l.* 10*s.* 10¾*d.* 170*l.* 10*s.* ½*d.* 100*l.* 10*s.* 6¾*d.* 4*l.* 16*s.* 6½*d.* 19*s.* ¾*d.* 37*l.* 11*s.* 11*d.* 600*l.* 10*s.* and 220*l.* 6½*d.* into one Sum.

(6) Add 276*l.* 17*s.* 16*l.* 10¾*d.* 269*l.* 11*s.* 11½*d.* 107*l.* 19*s.* 10*l.* 6*d.* 14*s.* 11*d.* 367*l.* 17*s.* 6¾*d.* 12*s.* 4½*d.* 20*l.* 10*s.* 6*d.* and 1000*l.* into one Sum.

Of WEIGHTS and MEASURES.

	oz.	*dwts.*	*grs.*		*lb.*	*oz.*	*dwts.*	*grs.*		℥	ʒ		*grs.*
(1)	27	11	20	(2)	27	10	17	11	(3)	11	2	1	17
	17	14	21		11	11	19	6		7	4	2	14
	46	17	11		4	6	14	17		4	1	1	19
	27	14	6		27	10	17	23		2	5	2	11
	4	9	17		17	7	11	17		10	1	2	16
	17	19	22		6	4	0	16		14	7	1	13
	27	17	16		17	11	18	15		11	4	2	11

(4)

lb.	℥	ʒ	℈	grs.
14	11	4	2	11
11	2	1	1	17
4	10	2	2	16
17	4	7	1	4
5	11		2	11
17	10	1	1	14
14	6	4	2	15

(5)

tons.	c.	qr.	lb.
14	17	2	14
417	11	1	21
24	6	3	27
219	14	1	14
36	17	2	26
11	14	1	14
6	11	2	19

(6)

lb.	oz.	drs.
14	11	14
17	14	11
5	6	12
21	4	15
36	13	11
14	7	10
6	11	6
4	4	12

(7)

yds.	qrs.	na.
14	2	3
276	1	0
37	3	2
4	1	1
110	0	0
17	3	2
13	2	3
106	1	2

(8)

Eng.	ells.	qrs.	na.
12	4	2	
27	0	0	
42	2	3	
146	3	2	
94	1	3	
62	2	1	
142	1	2	
41	2	3	

(9)

Fl.	ells.	qrs.	na.
17	2	1	
42	1	2	
146	2	1	
64	1	3	
72	2	1	
87	1	2	
100	0	0	
43	2	3	

(10)

lea.	m.	fur.	p.
12	1	7	14
27	1	4	27
141	2	6	36
84	0	7	39
100	1	4	11
36	2	5	13
4	0	0	24
120	2	6	6

(11)

yrds.	f.	in.	b.	c.
141	2	11	1	
27	1	4	1	
214	2	10	2	
76	0	11	0	
217	2	4	2	
96	1	11	1	
140	2	0	0	
60	0	10	1	

(12)

A.	r.	p.
210	2	27
74	3	14
142	1	37
47	2	4
149	0	27
34	3	36
8	1	11

(13)

tuns.	p.	hds.	ga.	qts.
12	1	1	14	2
14	1	1	27	3
10	1	0	61	1
6	1	1	42	2
2	0	0	26	3
13	1	1	4	2
6	0	0	36	3

(14)

punch.	ga.	qts.	pts.
14	14	2	1
7	32	3	1
24	51	2	1
14	14	1	1
49	36	3	1
37	17	1	1
8	62	3	1

(15)

tier.	gal.	pts.
12	24	7
41	41	4
3	26	2
6	14	5
27	39	6
19	14	4
21	34	3

(16)

ank.	gal.	pts.
10	7	4
14	9	7
27	4	2
4	6	0
11	5	3
2	3	5
17	2	6

(17)

A. hhds.	gal.	qts.
14	12	2
6	41	3
17	27	1
8	34	2
47	40	3
4	27	1
18	11	0

(18)

B. hhds.	gal.	pts.
24	51	7
14	17	4
6	8	6
14	12	0
9	47	4
34	36	5
17	11	2
4	29	7

(19)

A. bar.	kil.	fir.	ga.	pts.
14	1	1	4	7
27	1	0	7	4
19	1	1	6	5
6	0	1	5	3
31	1	0	3	2
6	0	1	1	6
10	1	1	4	3
6	1	0	6	0

(20)

B. fir.	gal.	qts.	pts.
14	8	2	1
9	7	1	0
10	4	3	1
6	6	2	1
27	5	1	0
8	2	3	1
41	1	0	1
6	3	1	1

(21)

qrs.	bu.	p.	gal.
14	7	2	0
27	4	3	1
142	6	1	1
19	4	2	0
4	6	3	1
127	4	1	0
41	1	2	1

(22)

cha.	bu.	p.
12	27	2
21	0	0
6	31	3
41	27	1
36	19	2
7	24	3
12	12	2

(23)

la.	w.	qrs.	bu.	p.
11	1	4	7	3
14	1	2	4	1
7	1	3	4	2
10	0	2	6	1
6	1	1	4	2
17	1	2	5	0
8	1	4	0	3

(24)

mo.	w.	d.	h.
11	2	4	21
24	3	6	14
12	1	0	23
31	2	5	0
14	1	1	11
6	3	6	17
9	2	1	20

(25)

d.	h.	m.	sec.
14	21	14	42
5	17	27	56
170	10	14	27
64	17	56	19
210	23	0	46
42	4	6	8
4	19	59	42

APPLICATION.

(1) HOW many Days are there from June 1 to Jan. 27 following?

(2) Suppose a man to be born in the year of our Lord 1772, in what year will he be 60 years of age?

(3) A Gentleman left his eldest daughter one thousand pounds more than the youngest, whose fortune was eleven thousand eleven hundred and eleven. What was the eldest daughter's fortune, and what did the father leave them?

(4) In the Bissextile or Leap Year, how many days in each month, and what is their sum?

(5) A. owed such a sum of money, that if he paid seventeen pounds seventeen shillings and sixpence, the remainder to pay would be eighty-two pounds two shillings and sixpence. Required the sum owed.

(6) A Privateer took a prize: the private men's share came to 474*l.* 17*s.* 11½*d.* and the officers received as much, besides 467*l.* unknown to the private men. How much did the officers receive?

(7) A Nobleman going out of town, is informed by his steward that his Corn-chandler's bill comes to 123*l.* 19*s*, His Brewer's to 41*l.* 10*s.* His Butcher's to 212*l.* To his Lordship's Baker is owing 24*l.* 6*s.* To his Tallow-chandler 13*l.* 8*s.* To his Tailor 137*l.* 9*s.* 9*d.* To his Draper 74*l.* 13*s.* 6*d.* His Coach-maker's demand was, 214*l.* 16*s.* 6*d.* His Wine-merchant's 68*l.* 12*s.* His Confectioner's 16*l.* 2*s.* His Rent 82 guineas, and his Servants' Wages for half a year came to 46*l.* 1*s.* What money must he send to his banker for, in case he would carry with him 50*l.* to defray his expenses on the road?

(8) A Corn-factor buys seventy quarters of oats, for 46*l.* 7*s.* 6*d.*, thirty-eight quarters of beans for 100*l.*, twelve quarters of pease, which cost 16*l.* 16*s.*, eighty-eight quarters of barley for 73*l.* 8*d.*, sixteen ditto of wheat for 56*l.* 9*s.* 10*d.*, and six quarters of rye for 4*l.* 4*s.* 6*d.* The water-carriage of all comes to 13*l.* 2*s.* 7*d.*, his riding charges to 1*l.* 13*s.*; and if he clear eighteen guineas by the bargain, what do his bills of parcels amount to?

(9) A Collector of cash has been out with bills, and gives an account that A. paid him 13*l.* and half-a-crown; B.

2*l.* 13*s.* C. 14*s.* and a groat. D. 1*l.* 9*s.* 8¼*d.* E. 11*l.* 6¾*d.* F. 17*s.* 6*d.* G. 12*s.* 6*d.* H. a pound and half a guinea. I. a moidore and 13*s.* K. two broad pieces of 23*s.* each, a Jacobus of 25*s.* and a shilling. L. nine pounds and a mark. M. 12*l.* 12*s.* N. a Bank note of 15*l.* and O. three crown pieces and an angel. What cash had he in charge?

(10) A. of Amsterdam is debtor to B. of Bristol, as per factor, 418*l.* 2*s.* 6*d.* for forty Cwt. of Cheshire cheese, 52*l.* 18*s.* for English broad-cloth, fifteen pieces, 317*l.* 12*s.* 10*d.* for 19 fodder of lead, 320*l.* for 12 tons of bar-iron, 173*l.* 3*d.* for eight tons of copper, 1110*l.* 10*s.* 1*d.* for his acceptance of a bill drawn, 88*l.* 14*s.* for another paid for honour, 50*l.* ten dozen of Morocco skins, 28*l.* 15*s.* 4*d.* paid convoys, insurance, and port charges, 43*l.* warehouse room, postage, sledage, boatage, and incidental charges, 5*l.* 5*s.* The factorage of all came to 112*l.* 6*d.* For what sum must B. draw to clear the account?

(11) In a gentleman's service of plate there are fourteen dishes, weighing 193 oz. 6 dwts. Plates thirty-six, weighing 421 oz. 11 dwts. four dozen of spoons, weighing 104 oz. 6 dwts. six salts chased, weighing 32 oz., knives and forks, weighing 83 oz. 9 dwts., four presenters, weighing 113 oz. 4 dwts. in mugs, tumblers, beakers, and other odd pieces, wt. 264 oz. 18 dwts. A silver tea-kettle and lamp, weighing 126 oz. 9 dwts., and the rest of that equipage 93 oz. 2 dwts. What quantity of plate had the butler under his care?

(12) A merchant buys four bags of Canterbury hops, No. 1. of which weighed 2 cwt. 2 qrs. 10 lb. No. 2. 2 cwt. 1 qr. 16lb. No. 3. 2 cwt. 24lb. No. 4. 1 qr. 16lb. besides a couple of pockets of ditto, that weighed 58½lb. each. How many hundred weight has he to pay carriage for, on bringing them to town?

(13) A gentleman at A. desired to know how far it was to E. and had the following answer, viz. From hence to B. is 39 m. 6 fur. thence to C. is 46 m. 24 p. thence to D. 60 m. 4 fur. 39 p. and thence to E. 37 m. 6 fur. What is the distance from A. to E.?

(14 A father was 28 years old (reckoning 13 months to 1 year, and 28 days to 1 month) when his eldest child was born; betwixt the eldest and second were 2 years,

10 months, and 16 days; betwixt the second and third were 1 year 11 months; betwixt the third and fourth were 3 years, 7 months, 25 days. When the fourth is 16 years, 9 months, 27 days, how old is the father?

QUESTIONS *for Exercise at leisure Hours.*

(15) How much is A., born 16 years ago, older than B. who will come into the world 14 years hence?

(16) A person was 17 years of age 29 years since, and he will be drowned 23 years hence. In what year of his age will this happen?

(17) A person said he had 20 children, and that it happened there was a year and a half between each of their ages; his eldest was born when he was 24 years old, and the age of the youngest is now twenty-one. What was the father's age?

(18) A sheepfold was robbed three nights successively; the first night half the sheep were stolen, and half a sheep more; the second night half the remainder were lost, and half a sheep more; the last night they took half what were left, and half a sheep more; by which time they were reduced to twenty. How many were there at first?

(19) Find how many years it was from the creation of Adam to the Universal Deluge in the days of Noah, called Noah's Flood; by the 5th chapter, and 6th verse of the 7th chapter of Genesis.

SUBTRACTION.

EXAMPLES of INTEGERS.

(1) From 476004 take 120706. (2) From 276000 take 106019. (3) From 40106 take 27109.

VIII. COMPOUND SUBTRACTION

TEACHETH to find the difference between any two sums of different denominations, as money, weights, measures, &c.

RULE.

Subtract as in Integers; only when the under number of any denomination is greater than that which stands over it, borrow so many of that denomination as make one of the

next superior; from which take the under number, and to the remainder add the upper number; which sum set down, remembering to carry or add one to the next higher denomination, before you subtract.

EXAMPLES of MONEY.

(1)

	l.	*s.*	*d.*
From	142	17	$11\frac{1}{2}$
Take	121	4	$6\frac{3}{4}$
Remains			

(2)

	l.	*s.*	*d.*
	210	10	10
	176	11	$11\frac{1}{4}$

(3)

	l.	*s.*	*d.*
Borrowed	264	0	0
Paid	176	12	$6\frac{1}{2}$
Rem. unpaid			

(4)

	l.	*s.*	*d.*
	153	14	$6\frac{1}{2}$
	76	0	$0\frac{3}{4}$

(5)

	l.	*s.*	*d.*
Lent	47	6	$0\frac{1}{2}$
Received	36	16	$0\frac{3}{4}$
Rem. unpaid			

(6)

	l.	*s.*	*d.*
	300	10	11
	196	17	$11\frac{3}{4}$

(7)

	l.	*s.*	*d.*
Borrowed	476	0	0
Paid at sundry times,	41	17	6
	127	10	11
	89	18	4
	94	17	10
	16	12	6
Paid in all			
Rem. unp.			

(8)

	l.	*s.*	*d.*
Lent	1476	14	6
Rec. at sundry times,	147	14	11
	376	19	6
	600	17	10
	276	14	6
	47	18	0
Received in all			
Rem. unpaid			

		l.	*s.*	*d.*
(9)	Borrowed	214	16	$4\frac{1}{2}$
		176	11	$11\frac{1}{4}$
		647	16	$4\frac{1}{2}$
	at sundry times,	84	12	$0\frac{1}{2}$
		317	16	$10\frac{1}{2}$
		176	15	4
		500	0	0
		147	17	6
		374	13	11
		7	19	6
Bor. in all				
Paid				
Rem. unp.				

		l.	*s.*	*d.*
	Paid	142	14	6
		176	17	$11\frac{1}{2}$
		67	19	6
	at sundry times,	476	0	0
		147	10	10
		67	18	8
		100	0	0
		427	18	$11\frac{1}{4}$
		176	6	4
		42	17	6
		17	10	0
	Paid in all			

Of WEIGHTS and MEASURES.

	(1) *lb.*	*oz.*	*dwts.*	*grs.*	(2) *oz.*	*dwts.*	*grs.*	(3) *lb.*	℥	ʒ	℈	*grs.*
Bought	14	10	12	17	16	10	21	17	11	4	0	11
Sold	11	11	14	21	12	17	14	14	10	6	2	17
Rem.												

	(4) *tons.*	*c.*	*qrs.*	*lb.*	(5) *lb.*	*oz.*	*drs.*	(6) *yds.*	*qrs.*	*na.*
From	14	11	2	17	140	10	14	141	2	3
Take	12	11	2	24	137	14	15	74	3	2
Rem.										

	(7) *Eng. ells.*	*qr.*	*na.*	(8) *Fl. ells.*	*qr.*	*na.*	(9) *lea.*	*m.*	*fur.*	*p.*
From	120	4	2	128	0	0	120	1	4	24
Take	117	4	3	69	2	3	80	2	7	37
Rem.										

	(10)				(11)			(12)				
	yds.	*f.*	*in.*	*b.c.*	*a.*	*r.*	*p.*	*tuns.*	*p.*	*hhd.*	*gal.*	*p.*
From	148	2	10	2	246	0	0	12	1	1	41	7
Take	97	2	11	2	178	2	24	11	1	0	61	6
Rem.												

	(13)				(14)			(15)		
	punch.	*gal.*	*qts.*	*pts.*	*tier.*	*gal.*	*pts.*	*ank.*	*gal.*	*pts.*
Bought	14	64	2	1	16	40	4	24	4	4
Sold	10	72	3	1	12	41	7	17	7	7
Rem. uns.										

	(16)			(17)			(18)		
	A.	*hhd.*	*gal.*	*qts.*	*B.*	*hhd.*	*gal.*	*pts.*	*B.*	*fir.*	*gal.*	*pts.*
From	24	41	2		12	46	4		14	4	4	
Take	17	47	3		10	51	7		10	8	6	
Rem.												

	(19)					(20)			(21)		
	A.	*ba.*	*fir.*	*gal.*	*qts.*	*pts.*	*qu.*	*bu.*	*p.*	*cha.*	*bu.*	*p.*
From	42	2	4	2	1		12	4	2	21	24	0
Take	14	3	7	2	1		5	7	3	14	34	2
Rem.												

	(22)					(23)				(24)			
	La.	*w.*	*q.*	*b.*	*p.*	*Mo.*	*w.*	*d.*	*h.*	*D.*	*h.*	*m.*	*sec.*
From	12	1	4	4	2	14	2	4	21	264	14	24	41
Take	8	0	4	7	3	11	2	6	22	107	21	41	56
Rem.													

APPLICATION.

(1) Suppose a person were born in the year of our Lord seventeen hundred and thirty-five, how old was he in the year 1789?

(2) There are two numbers, the greater is 102, and the lesser 72; what is their difference and sum?

(3) A. and B. having each a sum of money, A.'s sum, which is the greatest, is 74*l.* 17*s.* and the difference of their sums is 49*l.* 13*s.* 6*d.* I demand B.'s sum.

(4) Suppose I borrow 100*l.* and pay in part 41*l.* 17*s.* 6*d.* how much remains to pay?

(5) Suppose a gentleman has an estate of 600*l.* per annum, and he pays land-tax 140*l.*, also for repairs 94*l.* 17*s.* 6*d.* what is his net estate per annum?

(6) A person dying, left 13111*l.* 10*s.* 6*d.* between his son and daughter; the daughter was to have eleven thousand eleven hundred and eleven pounds. What was the son's fortune?

(7) A horse in his furniture is worth 35*l.* 10*s.*, out of it 12*l.* 12*s.* How much does the price of the furniture exceed that of the horse?

(8) A trader failing, was indebted to A. 71*l.* 12*s.* 6*d.*; to B. 34*l.* 9*s.* 9*d.*; to C. 16*l.* 8*s.* 8*d.*; to D. 44*l.*; to E. 66*l.* 7*s.* 6*d.*; to F. 11*l.* 2*s.* 3*d.*; to G. 19*l.* 19*s.*; and to H. a fine of 30 marks. At the time of this disaster he had by him in cash 3*l.* 13*s.* 6*d.*; in commodities he had 23*l.* 10*s.*; in household furniture, 13*l.* 8*s.* 6*d.*; in plate, 7*l.* 18*s.* 5*d.*; in a tenement, 56*l.* 15*s.*; in recoverable book debts, 87*l.* 13*s.* 10*d.* Supposing these things faithfully surrendered to his creditors, what will they then lose by him?

(9) A. gave a bond for 114*l.* 10*s.* the interest came to 19*l.* He then paid off 40 guineas, and gave a fresh bond for what was behind. By the time there was 13*l.* 4*s.* 8*d.* due on the second interest, he paid off 37*l.* 14*s.* 2*d.* more; took up the old bond, and signed a new one still for the residue; the principal again ran on till there was 9*l.* 11*s.* 3*d.* more due, and then he determined to take it up. What money had his creditor to receive?

(10) A chaise, horse, and harness, were all together valued at 50*l.* the horse in harness was worth 38*l.* 16*s.* 6*d.*, the chaise and harness were estimated at 13 guineas. Their several valuations are required.

(11) Received in lieu of two gold repeaters, sent to Jamaica in 1787, the five chests of indigo following; and on a like adventure in 1789, the subsequent five chests. The question is, how much indigo had I less the second time than the first?

Anno 1787	cwt.	qrs.	lb.		lb.	A. 1789.	cwt.	qrs.	lb.		lb.
No. 1	2	1	16	Tare	43		1	3	7	Tare	32
2	2	2	11		47		1	3	17		32
3	5	0	12		41		1	2	10		30
4	2	0	19		42		1	0	13		27
5	2	3	17		49		2	0	11		34

(12) A. B. and C. open an account with a banker, Jan. 11, 1806, and put into his hands, viz. A. 17 guineas, B. 34*l.* 11*s.* 6*d.*, C. 28*l.* 18*s.* 10*d.* On the 21st, A. withdrew 9*l.* 10*s.* and C. advanced 12*l.* and a crown. The 24th, B. called for 6*l.* 10*s.* The 30th, C. wanted 19*l.* 8*s.* 4*d.* On the 12th of Feb. B. deposited with him eleven carolus's, each 23*s.* and 3 moidores. On the 19th, A. sent for 5*l.* and a noble more; but on the 23d returned him 42*l.* On the 2d of March, C. paid in twenty guineas, and B. drew for six. The 14th B. sent in 17*l.* 8*s.* 8*d.* and the 17th A. had back 12*l.* 2*s.* 6*d.* On the 19th they sent for five guineas a man, and on the 24th they returned that sum, and ten marks a-piece more. How much did their said banker owe them jointly and separately at Lady-day.

(13) A merchant, taking an inventory of his capital, finds in his vaults 28 puncheons of brandy, which cost him 874*l.* 10*s.* 6*d.* Bourdeaux claret, 40 tuns, which stood him in 754*l.* 4*s.*, 22 lasts 4 bushels of corn in his granary, worth 675*l.* 17*s.* 3*d.*, with 2 lasts of canary seed, worth 113*l.* In his warehouse were ten casks of indigo, worth 632*l.* 12*s.* A parcel of saffron, worth 253*l.* 5*s.* W. P. of Stafford owed him 384*l.* 10*s.* In the hands of F. G. at Lynn, he had wines to the amount of 1011*l.* 10*s.* Pepper in the keeping of S. Q. of the Custom-house, value 1252*l.* 16*s.* 8*d.*, besides which R. O. owes him on bond 300*l.* and T. M. on note 260*l.* 14*s.* He has in India bonds to the value of 459*l.* and the interest of those securities made 25*l.* 14*s.* He had Bank stock to the value of 2134*l.* 4*s.* 6*d.* There lay in his banker's hands 1892*l.* 17*s.* 6*d.* He was at this time indebted to D. E. 713*l.* 13*s.*; to M. F. 352*l.* 10*s.* 8*d.*; to L. P. the foot of his account, 172 guineas; to J. B. on balance, 57*l.* 12*s.* 10*d.*; to an insurance, 190*l.* The present state of this person's fortune is required.

(14) A merchant at his commencing trade owed 280*l.* He had, in cash, commodities, the stocks, and good debts,

11505*l.* 10*s.* He cleared the first year, by commerce, 393*l.* 13*s.* 1*d.* What was his net balance at the year's end?

(15) Received from my factor at Alicant, on account of sales of tin, to the value of 197*l.* 12*s.* sterling; of bees'-wax to 71*l.* 7*s.* 6*d.*, of stockings to 47*l.* 3*s.* 6*d.*, of tobacco, the net proceeds whereof were 943*l.* 15*s.* 10*d.*, of cotton 123*l.* 3*s.* 7*d.* and of wheat to the amount of 116*l.* 5*s.* 6*d.* He at the same time advises, that he has, per order, shipped, for my account and risk, Alicant wines to the value of 226*l.* 16*s.* 6*d.* Figs, 150*l.* 11*s.* 3*d.* Fruit, 90 chests cost 104*l.* 6*d.* Olives, 136*l.* 10*s.* Oil, 193*l.* 17*s.* Raisins, 143*l.* 4*d.* and Spanish wool to the value of 75*l.* 13*s.* 8*d.* The commission of the whole consignment came to 71*l.* 18*s.* 11*d.* The question is, which of us is to draw for the difference, and how much?

(16) Jacob by contract was to serve Laban for his two daughters 14 years; and when he had accomplished 11 years, 11 months, 11 weeks, 11 days, 11 hours, and 11 minutes, pray how long had he to serve?

(17) W. X. Y. and Z. sent their money to the Bank, and drew upon it in the following manner, viz. June 4, 1805, Z. sent in 70*l.* 8*s.* Y. had 116*l.* 14*s.* 10*d.* remaining on balance, and on the 14th sent in 120*l.* more. W. paid in 47*l.* 18*s.* 2*d.* in cash, and delivered a Bank note for 200*l.* X. paid in a bill of exchange for 33*l.* 14*s.* 9*d.* and in cash made it up 100*l.* Y. on the 16th drew for 43*l.* 12*s.* 6*d.* and the 20th Z. for eleven guineas. W. on the 24th added 14*l.* 12*s.* 10*d.* and X. withdrew 47*l.* 10*s.* 8*d.* Y. on the 28th paid in 18*l.* 5*s.* and two days after drew for 88*l.* 13*s.* 4*d.* W. sent for 63 guineas on the 30th, and in five days after for 15*l.* 10*s.* 9*d.* more. Z., on the 7th of July, demanded 12*l.* 8*s.* 3*d.* and X. 7*l.* 3*s.* 1*d.* Z. on the 15th, remitted them 31*l.* 12*s.* 4*d.* and per assignment they received from him at the same time double that sum. Y. required 81*l.* 19*s.* 8*d.* on the 12th, and W. ten guineas. Y. three days after that sent in 42*l.* and W. 52*l.* On the 19th X. sent for 38*l.* 18*s.* 10*d.* and the 24th paid in 19 guineas. The question is, how stood these gentlemen's cash severally, and what money could they jointly raise?

QUESTIONS *for Exercise at leisure Hours.*

(18) Having a piece of ground 127 feet in front, let off to A. 57 feet, to build on at one end; and to B. at first 27½ feet, which he afterwards, by consent, extended to 42 feet. What ground was left me in the centre?

(19) If I am 42 years older than you now, what will be the difference of our ages 14 years after my decease, in case you should then survive?

(20) Of the noble family of Cornaro, the grandsire's age was 134 years, and he was 39 years older than the son, at the time when the son and father's ages together made 112 years. Distinguish their ages.

(21) B. was 14 years old when C. was 25. How old will C. be when B. comes to be 25?

(22) What is the difference between the ages of A. born in the year 1693, and B. that will be born 13 years hence, the question being put in the year 1789?

(23) When the air presses with its full weight, in very fair weather, it may be demonstrated, that there presses upon a human body about 33905 pounds of that fluid matter; and in foul weather, when the air is most light, but 30624 pounds. What difference of weight lies on such a body, in the two greatest alterations of the weather?

(24) Hipparchus and Archimedes of Syracuse, about 200 years before Christ, Posidonius 50 years before the said grand period, and Ptolemy 140 years after it, all advanced the science of astronomy. How long did each of these persons flourish before the year of Christ 1806?

(25) In the city of Pekin in China, is a bell weighing, it is said, 120000*lb*. At Nankin, in the same country, is another, weighing 50000*lb*. The first exceeds the great bell at Erford, in Upper Saxony, by 94600*lb*. How much then is the German bell inferior in weight to the second?

(26) Your grandfather, if living, is 119 years of age, your father actually 65; you are not so old as your grandsire by 83 years. What is the difference in years between your father and you.

(27) A snail, in getting up a May-pole only 20 feet high, was observed to climb eight feet every day; but every night it came down again four feet. In what time by this method did he reach the top of the pole?

(28) A. is 13 years younger than B. and 17 years older than C. who in the year 1747 was known to be 24 years of age. How old was each of these persons in 1784?

(29) A public edifice was finished towards the close of the 10th of King John, who began his reign 134 years after the conquest in 1066; and it stood till within 76 years of the peace of Utrecht, in 1713. Of what duration was it?

(30) A grant was made by the crown, anno 1239, which was forfeited 137 years before the revolution in 1688. How long did the same subsist?

(31) Moses was born anno Mundi 2433; Homer 832 years after him; Julius Cæsar lived 40 years before our Saviour, and Alexander 312 years before Cæsar; now, as Christ was incarnate 4000 years after the creation, the sum of the intervals between Homer and the three great personages last mentioned is required.

(32) The semidiameter of the earth's orbit, or annual path round the sun, in the centre of the system, is about 81,000,000 of miles, that of Venus 59,000,000: when they are both on the same side the sun, they are in Perigæo; when on different sides, in Apogæo. What is the difference of their distances in both these circumstances?

(33) B. was born 14 years after C. who came into the world 19 years before A. who was 23 years of age eight years ago. What then is the age of D. who is within 22 years of being as old as those three together?

(34) Arphaxad was born to Shem two years after the Deluge, and 500 before his father's death; but at 35 years of age he had Selah, who at 30 was father to Eber; who at 34 had Peleg, and he lived 430 years after that. The question is, whether Shem or Eber died the first? and, at nine score and fourteen years after the death of the longest liver, what interval might be wanting to complete the term of 1000 years after the Flood?

(35) K. is 19 years older than L. who was 27 years of age in the South-Sea year, 1720. How old was M. in 1740, who, in the year 1738, was within 24 years of being as old as both of them together?

(36) A. born A. D. 318, lived 207 years before B. who lived 104 years after C. who was successor to D. 84 years. E. was also 112 years after D. but predecessor to F. by 47 years. In what year did each of these gentlemen flourish?

(37) Sam was born 28 years before Toby, who died at 12, and lived 19 years after him. Rachel came to light when Sam was 16, and died 11 years before him. Joshua, when Rachel was 7 years, being himself then 14, went abroad, where he continued 9 years, and returning, survived Rachel 4 years. How old was each of these, and what is the sum of their ages?

(38) B. born anno 1108 lived 48 years before C. who was 113 years senior to D., and X. was 114 years before Y., who was 74 years after Z., born anno 1527. In what years were these men severally born?

(39) You were born 34 years after me. How old shall I be when you are 17? And how old will you be when I am 70 years of age?

(40) Five notable discoveries were made in 215 years' time, viz. 1. The invention of the Compass. 2. Gun-powder. 3. Printing. 4. The discovery of America. 5. Truth, in the Reformation. The last was brought about anno 1517; the third 77 years before; the second 42 years after the first, and the fourth 148 years after the second. The question is, in what year of Christ did each of these happen to be found?

(41) Three and thirty years before the Restoration in 1660, the crown granted demesnes to certain uses for 210 years then to come. The proprietor, in 1715, procured a reversionary grant for 99 years, to commence after the expiration of the first. In what year of Christ will the second term end?

(42) A. was born when B. was 18 years of age. How old will A. be when B. is 41? And what will be the age of B. when A. is 72?

(43) The building of Solomon's temple was in the year of the world 3000; Troy was by computation built 443 years before the temple, and 260 years before London. Now Carthage was built 113 years before Rome, founded 744 years before Christ, born anno Mundi 4000. Is London or Carthage the most ancient city, and how much?

(44) If the mean distance between the earth and sun be 81 millions of miles, and between the earth and moon 240 thousand, how far are those two luminaries asunder in an eclipse of the sun, when the moon is lineally between the earth and sun? and in another of the moon, when the earth is in a line between her and him?

(45) From the Creation to the Flood was 1656 years; thence to the building of Solomon's temple 1336 years; thence to Mahomet, who lived 622 years after Christ, 1630 years. In what year of the world was Christ then born?

(46) Seth was born when Adam was 130 years of age, and 800 years before Adam's death: Seth at the age of 105 years had Enos: he, at 90, was father to Cainan, who at 70 had Mahalaleel. This man at 65 begat Jared, who, at the age of 162 years, was father to Enoch: this patriarch at 65 years of age had Methuselah; and by the time he was 187 years of age, his son Lamech came into the world, who at 182 years old was father to Noah; and when Noah was 600 years old, the Flood swept away the bulk of mankind. In what year of the world did this happen, and how long after the death of Adam?

(47) Miss Kitty told her sister Charlotte, whose father had before left them twelve thousand twelve hundred pounds a piece, that their grandmother by will had raised her fortune to fifteen thousand pounds, and had made her own twenty thousand. What did the old lady leave between them?

(48) The powder-plot was discovered 88 years after the Reformation in 1517: The murder of King Charles the First was committed 43 years after that discovery: The accession of the Brunswick family to the crown was in 1714, just 54 years after the return of King Charles the Second, who had lived in exile ever since the death of his father Charles the First. How long was that?

(49) B., born 161 years ago, died when C. was 47 years of age, who it seems came into the world 180 years since, and out-lived B. 43 years. The sum of their ages is required.

(50) If Samson were born 17 years after Timothy, and Timothy 26 years before Jacob, who 28 years hence

will be just 50, in what year of Christ were they severally born; the question being proposed anno 1806?

(51) A. born 445 years before the year 1733, died anno 1362; B. born 37 years ago, will die 18 years hence; C. born 256 years ago, died 197 years since; D. born anno 1578, lived till within 75 years of the said 1733. The length of these people's lives is severally required.

(52) A. born anno 1441, lived till B. was seven years of age, which was 23 years before the Reformation in 1517: B. survived this remarkable æra just 49 years; C. born 9 years after the death of A. lived but till B. was 36 years of age. The sum of the ages of these three persons is required.

(53) A. born anno 1438, died at 48 years of age: B. died anno 1502, aged threescore and seventeen; C. in the year 1577, was 22 years of age, and survived that time 54 years; D., anno 1616, had lived just half his time, and died in 1648; E. was 13 years old at the death of D. and 14 years after that was father to F. who was 31 when his son G. was born, who at his grandsire's death was 7 years of age. The years of Christ, wherein these men were born, and the years wherein the first five of them died, are severally required.

(54) A. born 17 years after C. and 13 before B. died 42 years before King George the Second's inauguration in 1727, aged 47 years: C. died anno 1712, and B. exactly 6 years before him; D. born 23 years before C. died at 64; E. born 11 years after B.'s death, died 12 years after the year 1733; and F. born just in the midway of the interval between the births of A. and D. did not reach the time of E.'s death by 14 years. What is the sum of all their ages, and which of them lived longest?

MULTIPLICATION.

EXAMPLES of INTEGERS.

(1)	Multiply	14276084	by 4	(5)	Multiply	147624	by 69
(2)		20749509	9	(6)		42768	748
(3)		1204674	12	(7)		10646	5278
(4)		4074746	16	(8)		14276	39674

(9)	Multiply	3142708	by	467852
(10)		27680709		40700609
(11)		2142760		4100
(12)		21700		954000
(13)		46904		27
(14)		10709		336

CONTRACTIONS.

Case 1. When the Multiplier consists of the same Figures in all the places, (i. é.) all 9's or all 7's, &c. then, for each Figure in the Multiplier, annex a cipher to the Multiplicand, and, from the line thus produced, subtract the Multiplicand; and if the repeating Figure is 9, the Remainder will be the Product required; but if any other figure, multiply it into the ninth part of the Remainder; or, for the figure 3, take the third part of the Remainder; and for six, multiply the third part by 2, which will give the required number.

EXAMPLES.

(1)	Multiply	47627	by	9999
(2)		27464		11111
(3)		4674		2222
(4)		47694		7777
(5)		74760		3333
(6)		42763		6666

In many cases the work may be performed with more ease, likewise more concisely, than is usually practised.

EXAMPLES.

(7)	Multiply	4276	by	126
(8)		6946		486

IX. COMPOUND MULTIPLICATION

TEACHETH to multiply, by one common Multiplier, any sum or number consisting of different denominations.

Case 1. When the given quantity doth not exceed 12.

RULE.

1. Write the Multiplier, or given quantity, under the lowest denomination of the Multiplicand.
2. Multiply the number of the lowest denomination by the Multiplier, and divide that Product by as many of that as make one of the next higher denomination, the same

which you stopped at in Addition; set down the remainder underneath its own place, and add the quotient to the next superior denomination, as you multiply; in this manner proceed with all the other denominations to the highest.

EXAMPLES of MONEY.

		l.	*s.*	*d.*		*l.*	*s.*	*d.*		*l.*	*s.*	*d.*
(1)	Multiply	14	17	11	(2)	140	10	0½	(3)	17	6	4
				2				6				9
	Product											

(4) 4 Yards of cloth, at 17*s.* 6¼*d.* per yard.
(5) 5 Cwt. of cheese, at 3*l.* 0*s.* 6*d*, per cwt.
(6) 7 Ells of Holland, at 7*s.* 10*d.* per ell.
(7) 8 Pounds of tea, at 18*s.* 9½*d.* per lb.
(8) 9 Gallons of wine, at 12*s.* 8*d.* per gallon.
(9) 10 Ankers of brandy, at 2*l.* 6*s.* 4*d.* per anker.
(10) 11 Barrels of small beer, at 12*s.* 7*d.* per barrel.
(11) 12 Firkins of butter, at 1*l.* 17*s.* 6¾*d.* per firkin.

Case 2. When the given quantity exceeds 12, and is such a number that any two figures in the Multiplication Table, being multiplied together, will produce it.

RULE.

Multiply the given price by one of those numbers, and the product by the other, which will give the answer.

EXAMPLES.

(12) 14 Ounces of silver, at 6*s.* 7¾*d.* per oz.
(13) 18 lb. of sugar, at 10¼*d.* per lb.
(14) 27 Quarters of wheat, at 2*l.* 9*s.* 6*d.* per quarter.
(15) 30 Yards of German serge, at 4*s.* 11½*d.* per yard.
(16) 36 Stone of wool, at 10*s.* 8*d.* per stone.
(17) 45 Yards of tape, at 2½*d.* per yard.
(18) 50 Moidores, at 27*s.* each.
(19) 56 Yards of shalloon, at 2*s.* 7½*d.* per yard.
(20) 64 Firkins of butter, at 1*l.* 11*s.* per firkin.
(21) 72 Reams of paper, at 15*s.* 9*d.* per ream.
(22) 80 Yards of Yorkshire camlet, at 11¼*d.* per yard.
(23) 84 Gallons of oil, at 9*s.* per gallon.

(24) 96 Yards of Indian dimity, at 1*s.* 10¾*d.* per yard.
(25) 99 Yards of broad cloth, at 18*s.* 11½*d.* per yard.
(26) 100 Yards of cambric, at 11*s.* 10*d.* per yard.
(27) 120 Hundred of pens, at 1*s.* 6*d.* per hundred.
(28) 132 Deals, at 1*s.* 10*d.* each.
(29) 144 lb. of tobacco, at 1*s.* 7¾*d.* per lb.

Case 3. When the given quantity cannot be produced by the multiplication of two small numbers.

RULE.

Find the nearest number to its less, by which multiply as before; then, for what is wanting, multiply the price by that number, and add it to the last Product, and the Total will be the Answer.

EXAMPLES.

(30) 17 Cwt. of Malaga raisins, at 1*l.* 4*s.* 10½*d.* per cwt.
(31) 19 lb. of fine hyson tea, at 19*s.* 11¾*d.* per lb.
(32) 29 Yards of diaper, at 1*s.* 7½*d.* per yard.
(33) 30 Dozen pair of stockings, at 2*l.* 17*s.* 6*d.* per doz.
(34) 47 Yards of flowered linen, at 5*s.* 10*d.* per yard.
(35) 58 Ells of Holland, at 10*s.* 4½*d.* per ell.
(36) 67 Cwt. of tobacco, at 5*l.* 17*s.* per cwt.
(37) 75 Dozen of soap, at 6*s.* 4½*d.* per dozen.
(38) 86 Yards of green silk damask, at 19*s.* 1¾*d.* per yd.
(39) 106 Of Vyse's Tutor's Guide, at 3*s.* 6*d.* each.

Case 4. When the given quantity consists of ¼, ½, or ¾,

RULE.

Divide the upper line (the price of one) by 4 for ¼, by 2 for ½, and for ¾, by 2 first for ½, then divide that Quotient by 2, for ¼; add them to the Product, and the Sum will be the Answer required.

EXAMPLES.

(40) 35½ Tons of hay, at 3*l.* 6*d.* per ton.
(41) 76¾ Dozen of red port, at 1*l.* 12*s.* 10*d.* per dozen.
(42) 17¼ Barrels of ale, at 36*s.* 6½*d.* per barrel.
(43) 8½ Butts of beer, at 4*l.* 6*s.* 7*d.* per butt.
(44) 100½ Acres of land, at 26*l.* 17*s.* 6*d.* per acre.

This method of finding the value of any quantity of goods under 100, at any price per yd. lb. &c. is of excellent use to such as buy or sell by retail.

But for great quantities, there are other methods much better. (See PRACTICE.)

Case 5. Sometimes it may so happen, that your given quantity, though considerably great, may be wrought by the continual product of three numbers, as the following:

EXAMPLES.

(45) 112 Bushels of oats, at 1*s.* 10¾*d.* per bushel.
(46) 336 Yards of dowlas, at 2*s.* 5*d.* per yard.
(47) 350 Oz. of cloves, at 11¾*d.* per oz.

Of WEIGHTS and MEASURES.

(1) Multiply 14 lb. 10 oz. 0 dwts. 21 grs. by 4.
(2) ——— 17 tons, 17 cwt. 0 qr. 24 lb. by 2.
(3) ——— 14 cwt. 0 qr. 21 lb. 0 oz. 14 drs. by 7.
(4) ——— 10 lb. 6 ℥ 4 ʒ 1 ℈ 17 grs. by 9.
(5) ——— 127 yds. 0 qr. 3 na. by 12.
(6) ——— 40 ells. Eng. 4 qrs. 2 na. by 11.
(7) ——— 120 lea. 7 fur. 24 p. by 5.
(8) ——— 147 yds. 2 f. 11 in. 2 b. c. by 6.
(9) ——— 46 W. hhds. 47 gal. 7 pts. by 3.
(10) ——— 6 tuns, 1 p. 1 hhd. 46 gal. 3 qts. by 8.
(11) ——— 27 tier. 41 gal. 2 qts. by 6.
(12) ——— 4 B. hhds. 47 gal. 6 pts. by 9.
(13) ——— 10 A. hhds. 17 gal. 3 qts. 1 pt. by 4.
(14) ——— 12 B. bar. 2 fir. 7 gal. 7 pts. by 6.
(15) ——— 140 A. 2 r. 26 p. by 5.
(16) ——— 74 lasts, 7 qrs. 4 bu. 1 p. by 7.
(17) ——— 365 d. 5 h. 48 m. 57 sec. by 12.

APPPLICATION.

(1) What number, taken from the square of 54, will leave 19 times 46?
(2) Suppose 50 men to take a prize, and each man's share comes to 145*l.*, what is the value of the prize?
(3) What is the difference, and what the sum, of six dozen dozen, and half a dozen dozen?
(4) A certain island contains 52 counties, every county 42 parishes, every parish 246 houses, and every house 10 persons. I demand the number of parishes, houses, and persons, that are in the whole island?
(5) What difference is there between twice eight and twenty and twice twenty-eight: as also, between twice five and fifty, and twice fifty-five?

(6) By God's blessing upon a merchant's industry, in ten years' time he found himself possessed of 13,000*l.* It appeared from his books, that the last three years he had cleared 873*l.* a-year: the three preceding but 586*l.* a-year, and before that but 364*l.* a-year. What was the state of his fortune at every year's end that he continued in trade, and what had he to begin with?

(7) A robbery being committed on the highway, there was assessed on a certain hundred, in the county of S., the sum of 373*l.* 14*s.* 8*d.* The four parishes paid 37*l.* 16*s.* 4*d.* each, the four hamlets 28*l.* 3*s.* 10*d.* each, the four townships 19 guineas each. What was the deficiency?

(8) At Leicester and several other places they weigh their coals by a machine, in the nature of a steel-yard, waggon and all; three of these draughts together amount to 137 cwt. 2 qrs. 10 lb. and the tare or weight of the waggon was 13 cwt. 1 qr. How many coals had the customer to pay for?

(9) A person dying left his widow 1780*l.* and 1250*l.* to each of his four children, 30 guineas a piece to 15 of his poor relations, and 150*l.* to charities; he had been 25½ years in trade, and at an average had cleared 126*l.* a-year. What had he to begin with?

(10) Suppose a gentleman's income is 500*l.* per annum, and he expends daily 19*s.* 11*d.* What does he lay up at the year's end?

(11) If a gentleman expend daily 1*l.* 12*s.* 6*d.* and at the year's end lay up 294*l.* 12*s.* 6*d.* I demand his yearly income.

(12) The remainder of a division sum is 20, the quotient 423; the divisor is the sum of both, and 19 more. What was the number to be divided?

(13) Suppose that for a quarter's rent I paid in money seven guineas and sixpence, and was allowed for small repairs 18*s.* 6*d.* and for the king's tax 8*s.* 9*d.* What did my tenement go at a-year?

(14) A person dying left his widow the use of 5000*l.* To a charity he bequeathed 846*l.* 10*s.* To each of his three nephews 1230*l.* To each of his four nieces 1050*l.* To 20 poor house-keepers five guineas each, and 200 guineas to his executor. What must he have died possessed of?

(15) A gentleman gave his daughter to her portion a scrutoir, in which were twelve drawers; in each of these were six divisions, and in each division there were 100*l.* a moidore, and half-a-guinea. What was the young lady's fortune?

QUESTIONS *for Exercise at leisure Hours.*

(16) The silk mill at Derby contains 26586 wheels, and 97746 movements, which wind off or throw 73726 yards of silk every time the great water-wheel, which gives motion to all the rest, goes about, which is three times in a minute. The question is, how many yards of silk may be thrown by this machine in a day, reckoning ten hours to a day's work? and how many in the compass of a year, deducting for Sundays and great holidays, 63 days; provided no part of it stands still?

(17) Trajan's bridge over the Danube is said to have had 20 piers to support the arches, every pier being 60 feet thick, and some of them were 150 feet above the bed of the river; they were also 170 feet asunder. Pray what was the width of the river in that place? and how much did it exceed the length of Westminster bridge, which is about 1200 feet from shore to shore, and is supported by 11 piers, making the number of arches 12?

(18) There are two numbers, the less is 187, their difference 34. Required the square of their product, ditto of their sum, and difference and sum of those squares.

(19) There are two numbers, the greater of them is 73 times 109; and their difference 17 times 28. I demand their sum and product.

(20) In the partition of lands in an American settlement, A. had 757 acres allotted to him; B. had 2104 acres; C. 16410; D. 12881; E. 11008; F. 9843; H. 13800; and I. 8818 acres. Now, how many acres did the settlement contain, since the allotments made as above want 416 acres of one-fifth of the whole?

(21) How many different ways can four common dice come up at one throw?—Note, One may come up six ways.

(22) In a company S. had 3*l.* 17*s.* 2*d.* more than T. who had six guineas less than R. who had within 16*s.* 8*d.* of as much as W. who was known to have 100 guineas, wanting ten marks of 13*s.* 4*d.* each. What money had they among them?

DIVISION.

EXAMPLES of INTEGERS.

(1)	Divide	14076893	by	4
(2)	———	30742165	by	12
(3)	———	2410296	by	89
(4)	———	98420649	by	576
(5)	———	308763705	by	3029
(6)	———	16221212499	by	46058
(7)	———	51799555	by	127345
(8)	———	78855994985	by	3090807
(9)	———	1276421427	by	3700
(10)	———	4074964478	by	827000
(11)	———	24769851400	by	9300
(12)	———	67402	by	14
(13)	———	701747	by	63
(14)	———	4170642	by	112
(15)	———	2741724	by	336

CONTRACTIONS.

When the divisor consists of the same figure in all the places; that is, all 9's or all 7's, &c. annex as many ciphers to unit or 1 as there are 9's or 7's, &c. in the given divisor, for a new divisor; and, if the repeating figure be 9, divide the dividend by that divisor, and do the same with the quotient, till you get 0 for an integral quotient; then add all the overplusses together, and divide that sum by the given divisor, the overplus thence arising is that required; and the sum of all the integral quotients is the quotient required. For any other figure, divide nine times the dividend so, and the integral quotient by the repeating figure. This gives the true integral quotient; and if the ninth part of the first overplus be added to the second, repeated as the given figure, the sum will be the true overplus.

EXAMPLES.

(16) Divide 4077823 by 999
(17) ——— 2692464 by 1111
(18) ——— 4769042 by 7777
(19) ——— 109494 by 6666

X. COMPOUND DIVISION

TEACHETH to divide, by one common divisor, either a simple or compound number into any proposed number of equal parts, whereof each shall be a compound number.

Case 1. When the divisor does not exceed 12.

RULE.

1. Place the divisor and dividend as integers.
2. Write their quotas under each respective dividend.
3. But if there be a remainder after dividing any of the denominations except the least, you must find how many of the next lower denomination it is equal to, by multiplying it by as many of the next less as make one of that, which add to the next, if any, and divide as before.

EXAMPLES in MONEY.

		l.	*s.*	*d.*			*l.*	*s.*	*d.*
(1)	2)	14	16	$10\frac{1}{2}$	(2)	7)	267	0	0

(3)	8)	171	11	4	(4)	9)	317	0	6	(5)	11)	6·6	$11\frac{1}{2}$

(6) Bought 3 cwt. of cheese, for which I gave 7*l.* 11*s.* 6*d.* At what rate did I give per cwt.?
(7) If 10 dozen of candles cost 3*l.* 17*s.* 1*d.* what costs one dozen?
(8) Suppose I give my servant fourteen guineas per year, what do his monthly wages come to?

Case 2. When the divisor exceeds 12, and is such a number that any two figures in the Multiplication Table being multiplied together, will produce it.

RULE.

Divide by component parts, as in Sect. V. Case 4.

EXAMPLES.

(9) Divide 45*l.* 12*s.* 8*d.* into 16 equal parts.
(10) Divide 3*l.* 13*s.* equally amongst 24 persons.
(11) What is cloth per yard, when 36 yards cost 64*l.* 19*s.*?
(12) What is tobacco per cwt. if 42 cwt. cost 190*l.* 4*s.* 6*d.*?
(13) Bought 48 yards of broad cloth for 37*l.* 14*s.* 8*d.* I desire to know at what rate I gave per yard?
(14) Suppose a man spends 78*l.* 16*s.* 8*d.* in eight months' time, what is that per week?
(15) A prize of 4567*l.* 0*s.* 10*d.* is to be equally divided amongst 55 persons. What is each man's share?
(16) What is tea per cwt. when 63 cwt. cost 264*l.* 12*s.*?
(17) If 72 oz. of silver cost 18 guineas, what is it per oz.?
(18) Suppose I have 81 cwt. of cheese, which cost me 121*l.* 12*s.* 6*d.* at what rate did I buy per cwt.?
(19) Divide 174*l.* 1*s.* 8*d.* equally amongst 120 sailors.

Case 3. When the divisor cannot be produced by the multiplication of two small numbers, divide as in Sect. IV. Case 2.

EXAMPLES.

(20) Divide 214*l.* 17*s.* 9¾*d.* equally among 17 persons.
(21) Divide 476*l.* amongst 145 people.

The following Examples require three Divisions:

(22) I gave 30*l.* 2*s.* for 112 yards of cambric. At what rate was that per yard?
(23) Divide 1009*l.* equally amongst 350 persons.
(24) Suppose the clothing of 224 charity children comes to 610*l.* 8*s.* what is the expense of each?
(25) Divide 1426*l.* equally among 640 persons.

Case 4. If the given quantity, or divisor, consist of $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$.

RULE.

Multiply the given quantity by 4, adding to the product 1 for $\frac{1}{4}$, 2 for $\frac{1}{2}$, 3 $\frac{3}{4}$; and it will give the divisor, which divide with, as before; and the quotient multiplied by 4 will give the answer.

EXAMPLES.

(26) Suppose I gave for 6¾ yards of cambric 12*l.* 12*s.* 11*d.* at what rate did I buy it per yard?

(27) Suppose a person in trade to clear 1061*l.* 8*s.* 9¾*d.* equally in 10½ years, what was his yearly increase of fortune?

(28) Suppose another to clear 450*l.* 13*s.* 11½*d.* equally in 8¾ years, what was his yearly profit?

Of WEIGHTS and MEASURES.

EXAMPLES.

(1) Divide 8 lb. 1 oz. 15 dwts. 8 grs. by 2.
(2) ——— 24 tons, 14 cwt. 0 qr. 14 lb. by 3.
(3) ——— 17 cwt. 2 qrs. 27 lb. 14 oz. 15 drs. by 4.
(4) ——— 4 lb. 11 ℥. 4 ʒ. 1 ℈. 12 grs. by 5.
(5) ——— 214 yds. 3 qrs. 2 na. by 9.
(6) ——— 120 ells. Eng. 4 qrs. by 8.
(7) ——— 12 lea. 2 m. 0 fur. 26 p. by 9.
(8) ——— 147 yds. 2 f. 11 in. 2 b.c. by 10.
(9) ——— 24 W. hhds. 57 gal. by 11.
(10) ——— 10 tuns, 1 p. 1 hhd. 60 gal. 3 qts. by 8.
(11) ——— 16 tier. 20 gal. 7 pts. by 6.
(12) ——— 76 A. hhds. 27 gal. by 5.
(13) ——— 12. B. hhds. 49 gal. 2 qts. by 4.
(14) ——— 61 B. bar. 2 fir. 6 gal. by 3.
(15) ——— 140 A. 2 r. 26 p. by 12
(16) ——— 60 lasts, 6 qrs. 7 bu. 2 pks. by 7.
(17) ——— 146 days, 23 h. 24 m. 56 sec. by 6.

APPLICATION.

(1) An army of 10000 men having plundered a city, took 220000*l.* What was each man's share?

(2) A certain man intending to go a journey of 336 miles, and to complete the same in twelve days; it is required how many miles he must travel each day?

(3) What number, added to the forty-third part of 4429, will make the sum of 240?

(4) What number, deducted from the twenty-sixth part of 2262, will leave the eighty-seventh part of the same?

(5) What number, multiplied by 72084, will produce 5190048?

(6) What number, divided by 419844, will quote 9494, and leave just a third part of the divisor remaining?

(7) The sum of two numbers is 360, the less is 144. What is their difference, product, and larger quote?

(8) The Spectator's Club of fat people, though it consisted but of 15 persons, is said (No. 9) to weigh no less than 3 tons. How much, on an equality, was that per man?

(9) What number is that, from which if you deduct the 25th part of 22525, and to the remainder add the 16th part of 9696, the sum will be 1440?

(10) What number, multiplied by 57, will produce just what 134 multiplied by 71 will do?

(11) Subtract 30079 out of fourscore and thirteen millions, as often as it can be found, and say what the last remainder exceeds or falls short of 21180?

(12) A gentleman at his death left his eldest son once and a half what he allotted his daughter, and to the young lady 1389*l.* less than her mother, to whom he bequeathed four times what he left towards the endowment of Hertford College, Oxon, viz. 1640 guineas. I require what he intended for his youngest son, who claimed under the will half as much as his mother and sister? How much less than 30000*l.* did the testator die worth, after his debts and funeral expenses, being 988*l.* 10*s.*, were paid?

(13) My purse and money, quoth Dick, are worth 12*s.* 8*d.* but the money is worth seven times the purse. Pray what was there in it?

(14) A young fellow owed his guardian 74*l.* 18*s.* 2*d.* on balance. He paid off 41*l.* 14*s.* 8*d.* and then declared his sister owed the gentleman half as much again as himself. On hearing this, she paid off in part 13*l.* 12*s.* 10*d.* and gave out that her uncle William was not then less in arrears than her brother and she together. The uncle hereupon paid in 24*l.* 7*s.* 3*d.* and then the uncle's brother, who, by the by, was not the uncle of those children, for 150*l.* undertook to set them all clear, and had, he said, 35*l.* 15*s.* 5*d.* to spare. Can that be true?

(15) A dealer bought two lots of snuff that together weighed 9 cwt. 3 qrs. 16 lb. for 97*l.* 17*s.* 6*d.* Their difference in point of weight was 1 cwt. 2 qrs. 16 lb. and of price 8*l.* 13*s.* 3*d.* Their respective weights and values are required.

(16) A father left among seven sons and a daughter an estate consisting of 10000*l.* in cash, with eight bills, each of 44*l.* 10*s.* 6*d.* He ordered 36*l.* to be bestowed upon his burial, and his debts to be paid, amounting to 260*l.* then his free estate to be divided in this manner, viz. the daughter to have the ninth part, and the seven sons to have equal shares. What is the daughter's part, and also what is the share of each son?

QUESTIONS *for Exercise at leisure Hours.*

(17) I would plant 2072 elms, in 14 rows, twenty-five feet asunder. How long must the grove be?

(18) A brigade of horse, consisting of 384 men, is to be formed into a long square, having 32 men in front. How many ranks will there be?

(19) Divide 1000 crowns betwixt A. B. and C. in such a manner that A. may have 129 more than B. and B. 178 less than C.

(20) Part 250*l.* Give A. 37 more than B. and let C. have 28 fewer.

(21) Six of the female cricketers that played lately in the Artillery-ground, fetched in company strokes as follow, viz. A. B. C. D. E. 207. A. C. D. E. F. 213. A. B. D. E. F. 189. A. B. C. E. F. 234. A. B. D. C. F. 222. B. C. D. E. F. 250. How many did they fetch on the other side, since these six persons wanted but fourscore and 13 notches to decide the game?

(22) In order to raise a joint stock of 10000*l.* L. M. and N. together subscribed 8500*l.* and O. the rest. Now M. and N. are known together to have set their hands to 6050*l.* and N. has been heard to say that he had undertaken for 420*l.* more than M. What did each proprietor advance?

(23) There are two numbers, whose product is 1610; the greater is 46. What is their sum, difference, and quotas; what is the sum of their squares; and what is the cube of their difference?

(24) There are other two numbers, the greater 7050, which, divided by the less, quotes 94. What is the difference of their squares? and what the square of the product of their sum and difference?

(25) What difference will there be to the proprietors of an aqueduct, between doubling an expense, and halving a profit?

(26) Part 1500 acres of land; give B. 72 more than A. and C. 112 more than B.

(27) One of the smarts in the Accomptant's Office making his addresses in an old lady's family, who had five fine daughters, she told him their father had made a whimsical will, which might not soon be settled in Chancery; and till then he must refrain his visits. The young gentleman undertook to unravel the will, which imported that the first four of her girls' fortunes were together to make 25000*l*, the four last 33000*l*. the three last with the first, 30000*l*. the three first, with the last, were to make 28000*l*. and the two last with the two first 32000*l*. "Now, sir," said the old lady, "if you can make appear what each is to have, and as you like, seemingly, my third daughter, Charlotte, I am sure she will make you a good wife, and you are welcome." What was Miss Charlotte's fortune?

(28) By selling 240 oranges at five for 2*d*. 120 of which cost me two a penny, and the other half three a penny, I evidently lose a groat. Pray how comes that about?

(29) A. B. and C. play in concert at Hazard; and at making up accounts, it appears that A. and B. together brought off 13*l*. 10*s*. B. and C. together 12*l*. 12*s*. and A. and C. together won 11*l*. 16*s*. 6*d*. What did they severally get?

(30) Four persons advanced in trade as follows, viz. W. X. and Y. raised 350*l*. 10*s*. W. X. and Z. 344*l*. 10*s*. X. Y. and Z. made up together 400*l*. and W. Y. and Z. contributed 378*l*. 4*s*. In the conclusion they parted with their joint property for 450 guineas. What did they gain or lose by their adventure?

(31) A tradesman increased his estate annually a third part, abating 100*l*. which he usually spent in his family, and at the end of 3¾ years found that his net

estate amounted to 3179*l.* 11*s.* 8*d.* Pray what had he at first?

(32) Ten pounds a quarter are allowed to five auditors of a fire-office. They attend about seven times in the quarter, and the absentees' money is always divided equally among such as do attend. A. and B. on these occasions never miss; C. and D. are generally twice in a quarter absent, and E. only once. At the payment, what has each man to receive?

(33) Suppose a maid carrying apples to market were met by three boys, and that the first took half what she had, but returned ten; that the second took one-third, but returned two; lastly, the third took away half those she had left, but returned one, and when she had got clear she had 12 apples left. What number of apples had she at first?

XI. REDUCTION.

IN this and all the following rules, all great names are brought into small by Multiplication; on the contrary, all small names into great by Division. See Sect. VI. page 15.

EXAMPLES in MONEY.

(1) In 130*l.* how many shillings, pence, and farthings?
(2) How many pence, shillings, and pounds, are in 24000 farthings?
(3) In 80*l.* 15*s.* 11¾*d.* how many farthings?
(4) Reduce 16921 farthings to pounds.
(5) Reduce 110*l.* 0*s.* 6½*d.* to halfpence.
(6) How many pounds, &c. are there in 20553 halfpence?
(7) In 107*l.* 10*s.* 8*d.* how many two-pences?
(8) Reduce 5348 two-pences to pounds.
(9) Reduce 6*l.* 17*s.* to three-pences.
(10) In 2782 three-pences, how many pounds, &c.?
(11) In 10*l.* 10*s.* 8*d.* how many four-pences?
(12) Reduce 38859 four-pences to pounds.
(13) How many six-pences are there in 200*l.* 17*s.*?
(14) Reduce 795 six-pences to pounds, &c.
(15) In 21 guineas, how many shillings, pence, and farthings?
(16) How many guineas in 24192 farthings?

(17) In 12 moidores how many farthings?
(18) How many moidores are there in 3240 pence?
(19) In 30*l.* how many crowns, half-crowns, and pence?
(20) Reduce 20160 pence to half crowns, crowns, and pounds.
(21) In 25 crowns, how many shillings, groats, and pence?
(22) Reduce 25200 pence to groats, shillings, and crowns.
(23) In 25*l.* how many shillings, crowns, and pence?
(24) How many shillings and pounds in 80 crowns?
(25) How many crowns, half-crowns, and shillings, are in 213*l.* 15*s.* 6*d.* and of each an equal number?
(26) In 120*l.* how many half-crowns, crowns, groats, and shillings?
(27) In 36 crowns, as many half-crowns, shillings, and groats, how many pounds?
(28) Reduce 470*l.* 17*s.* to shillings and moidores.
(29) Reduce 240 guineas to shillings, crowns, and pounds.
(30) In 21 purses, each purse with 21 guineas, a crown, and a moidore in, what sterling do they contain?

COINS.

Case 1. To reduce Foreign and English coins to pounds sterling.

RULE.

Multiply the given number of pieces, by the shillings, six-pences, four-pences, three-pences, two-pences, pence, or half-pence, &c. that are in one piece; and the product will be accordingly either shillings, six-pences, &c. which bring into pounds sterling.

EXAMPLES.

(31) How many pounds sterling are there in 1178 dollars, at 4*s.* 3*d.* each?
(32) In 470 pistoles, each 17*s.* 6*d.* how many pounds sterling?

Case 2. To reduce pounds sterling into Foreign and English coin, &c.

RULE.

Reduce the given pounds sterling, and the given coin, into one name; that is, if you can reduce them both into shillings, six-pences, four-pences, or three-pences, &c. do so; then divide one by the other, and the quotient will be the answer.

EXAMPLES.

(33) A merchant is to pay 249*l.* 7*s.* 6*d.* With how many quarter-guineas can he do it?

(34) In 387*l.* 18*s.* 4*d.* how many florins at 3*s.* 2*d.* each?

Case 3. To reduce one kind of coin into another kind of coin.

RULE.

Reduce both coins into the same denomination, and then divide one by the other.

EXAMPLES.

(35) How many crowns of 5*s.* 4*d.* each, are in 474 pistoles of 18*s.* 6*d.* each?

(36) How many guineas are equal in value to 1240 moidores?

Of WEIGHTS and MEASURES.

(1) In 14 lb. of silver, how many ounces, penny-weights, and grains?

(2) How many lb. of silver are there in 138240 grains?

(3) In 19 lb. 10 oz. 17 dwts. 22 grs. how many grains?

(4) Reduce 74342 grs. to pounds.

(5) In 4 ingots of silver, each weighing 4 lb. 6 oz. 22 grs. how many grains?

(6) How many ingots, of 6 lb. 11 oz. 14 dwts. each, are there in 241056 grains?

(7) How many lb. of silver are there in one dozen of dishes, each weighing 25 oz. 15 dwts. and one dozen of plates, each weighing 15 oz. 15 dwts. 22 grs.?

(8) A gentleman sent 455 oz. 1 dwt. 16 grs. of old plate, to his silversmith, with orders to make it into the following articles, viz. Punch bowls, each 24 oz. 4 dwts. Tan-

kards, each 11 oz. 14 dwts. Tea-pots, each 10 oz 10 dwts. Lamps, each 20 oz. 17 dwts. 21 grs. Plates, 127 oz. 11 dwts. per dozen. Spoons, 86 oz. 17 dwts. 23 grs. per dozen. How many of each must he make, supposing for every dozen of plates and spoons, he is to make one of each of the other?

(9) In 4 lb. 10 oz. 4 drs. 1 scr. 12 grs. how many grains?

(10) Reduce 59934 grs. to ounces and pounds.

(11) In 6 tons, how many cwt. qrs. and lbs.?

(12) How many tons in 26880 lb.?

(13) Reduce 74 cwt. 2 qrs. 16 lb. 7 drs. to drams.

(14) In 29768 oz. how many hundred weight?

(15) Reduce 67 lb. 12 oz. 15 drs. to drams.

(16) In 6 hhds. of tobacco, each weighing net 6 cwt. 3 qrs. 27 lb. how many pounds?

(17) How many hhds. of sugar, each weighing 11¼ cwt. are there in 12880 lb.?

(18) In 507 cwt. of lead, how many fother?

(19) Out of 12 cwt. 3 qrs. 12 lb. of tea, how many canisters can I fill, each canister holding 12 lb.?

(20) How many parcels, each 126½ lb. can I have out of one hhd. of sugar weighing net 8¾ cwt.?

(21) How many parcels of 6 lb. 8 lb. 12 lb. and 16 lb. can a grocer have out of two hhds. of tobacco, each weighing net 4 cwt. 3 qrs. 24 lb. and to have of each a like number?

(22) Reduce 24 great pounds, at 24 oz. each, to common pounds, at 16.

(23) In 120 common pounds, how many great pounds?

(24) In 27 yds. 3 qrs. of cloth, how many nails?

(25) How many yards in 352 nails?

(26) Reduce 30 ells English, 4 qrs. 3 na. to nails.

(27) In 569 nails, how many Fl. ells?

(28) In 14 pieces of cloth, each 24 yds. how many nails?

(29) Reduce 24768 nails to pieces, each 12 yards.

(30) In 12 pieces of cloth, each containing 20 Flemish ells, how many ells English?

(31) How many pieces of cloth, each 24 ells Flemish, are there in 227 yards?

(32) In 4 bales of cloth, each 12 pieces, and each piece 24 ells Fl. how many ells English?

(33) In 60 miles, how many furlongs and poles?

(34) Reduce 12800 poles to miles.

(35) In 16 miles, how many feet, inches, and barley-corns?
(36) Reduce 2280060 barley-corns to miles.
(37) How many barley-corns will reach from London to Newcastle-upon-Tyne, being 276 miles?
(38) How many times does a wheel, which is 18½ feet in circumference, turn between London and York, being 197 miles?
(39) How many barley-corns will reach round the terrestrial globe, which is 360 degrees, and each degree 69½ miles?
(40) In 64 acres of land, how many roods and poles?
(41) Reduce 21760 poles to acres.
(42) A common field, containing 774 acres, is to be divided into shares of 270 perches each. How many shares doth the whole contain?
(43) A person rents a farm, which contains 200 acres of land; but he is to till no more than 96½ acres. I desire to know how many perches there are in the remainder.
(44) In 12 tierces of wine, how many gallons and pints?
(45) How many tierces in 6048 pints?
(46) In 4 hhds. 42 gal. 2 qts. of wine, how many quarts?
(47) How many hhds. of wine in 5746 pints?
(48) A gentleman ordered his butler to bottle off a pipe of red port into quart bottles. How many dozen will the said pipe fill?
(49) In a tun of oil, how many quarts, pints, and half-pints, and of each an equal number?
(50) How many pipes, puncheons, hogsheads, and tierces, and of each a like number, are there in 1890 gallons?
(51) In 12 barrels of ale, how many gallons and pints?
(52) How many barrels of ale in 1704 pints?
(53) In 6 bar. 2 fir. 7 gal. of beer, how many gallons?
(54) In 10 hhds. 42 gal. 4 pts. of ale, how many pints?
(55) In 2017 qts. of ale, how many hdds.?
(56) In 12 hhds. of beer, how many barrels?
(57) In 18 barrels of ale, how many hhds.?
(58) In 4 tuns, 1 b. 1 hhd. 40 gal. of beer, how many hhds. bar. and fir. and of each a like number?
(59) Reduce 24 qrs. of wheat to bushels, pecks, and gal.
(60) In 3360 gallons of corn, how many quarters?
(61) How many quarters and bushels are there in 42 la. 4 qrs. 7 bu. of wheat?
(62) In 40 chaldrons of coals, how many bush. and pecks?
(63) How many chaldrons of coals are there in 4762 bush.?

(64) In 47 ch. 30 bush. of coals, how many sacks, each 3 bushels?

(65) How many chaldrons of coals are there in 6450 sacks, each three bushels?

(66) How many minutes are there in a Julian year?

(67) Reduce 2073600 seconds to days.

(68) In a lunar month, or 27 d. 7 h. 43 m. 5. sec. how many seconds?

(69) How many thirds are there in a solar year?

(70) In 31557600 seconds, how many days?

(71) How many days from the birth of our Saviour to Christmas, 1810 (allowing Julian years)?

(72) Suppose London was built 1108 years before the birth of our Saviour; how many days from that time to Christmas 1789, allowing the year as before?

XII. The RULE of THREE DIRECT

TEACHETH by three numbers given to find a fourth, in such proportion to the third, as the second is to the first: for which reason it is termed the RULE of PROPORTION, as it is called the RULE of THREE from its having three numbers given; and because of its excellent and extensive use in Arithmetic, it is often named the GOLDEN RULE.

To perform which, observe the following

RULE.

1. State or place the numbers in such order, that the first and third terms be of the same kind; and the second of the same with the number required.
2. If your first and third terms consist of different denominations, reduce them into one, and the second into the lowest name mentioned.
3. Multiply the second and third terms together, and divide that product by the first. The quotient will be the answer to the question, in the same denomination you left your second term in.
4. If there happen to be a remainder after the division, reduce it into the next denomination below the last quotient; and divide by the same divisor; the quotient will be so many of the said next name; proceed in this manner to the least name; and all the quotients together will be the answer. Or, when convenient, multiply and divide as in Sects. 9 and 10.

EXAMPLES.

(1) If three yards of cloth cost 18*s.* what will be the value of 17 yards, at the same rate?

(2) If 2 lb. of sugar cost 1*s* 6½*d.* what will 24 lb. of the same cost?

(3) If 4 lb. of candles cost 2*s.* 6*d.* what will 6 dozen cost at the same rate?

(4) If 1 cwt. of cheese cost 26*s.* what will 40 cwt. of the same come to?

(5) Suppose I give 7½*d.* for 1 oz. of coffee, what must I pay for 1 cwt.

(6) Bought 36 oz. of silver at the rate of 5*s.* 4*d.* per ounce. What does the whole come to?

(7) If I buy 12 pieces of cloth, and each piece contains 30 yards, at 17*s.* 6*d.* per yard; what is the value of the whole?

(8) Bought 1 cwt. of tea, for which I was to give at the rate of 7*s.* 9*d.* per lb. What does the whole stand me in?

(9) A grocer bought 2 cwt. 1 qr. 14 lb. weight of cloves, which cost him 34*l.* 6*s.* and he would gain 6*l.* by the bargain. At what rate must he sell them per lb.?

(10) Suppose I have by me 200 yards of cambric, which cost 90*l.* but some damage having happened to it, I am willing to lose 7*l.* 10*s.* by the whole; at what rate then must I sell it per ell English?

(11) If 9 dozen pounds of candles cost 2*l.* 5*s.* what will 4 lb. of the same cost?

(12) A grocer bought 4 hhds. of sugar, each weighing net 12 cwt. 2 qrs. 24 lb. and gave after the rate of 6½*d.* per lb. I demand what the 4 hhds. came to?

(13) A merchant at London buys 46 tuns of port wine, which cost him 579*l.* 12*s.* the freight thereof from Oporto to London costs 46*l.* the loading and unloading 6*l.* custom 10*l.* the charge of the cellar 4*l.* and he would gain 360*l.* by the bargain. A gentleman comes to him and demands the price of 26 tuns of the said wine. Quere, what must he give?

(14) A factor bought of a farmer 12 cwt. 2 qrs. 14 lb. of cheese, and was to give 32*s.* 6*d.* per cwt. What must the farmer receive for his cheese?

(15) If 1 cwt. of tea cost 89*l.* 16*s.* 4*d.* at what rate must it be sold per lb. to lose by the whole 12*l.*?

(16) If in four months I spend as much as I gain in three, how much do I lay by at the year's end, if I gain every six months 185*l.* 5*s.* 6*d.*?

(17) How many dozen pair of gloves, at 16*d.* per pair, will pay for 36 dozen and 8 pair of stockings, at 4*s.* 6*d.* per pair?

(18) Bought a parcel of cloth at the rate of 6*s.* 6*d.* for every two yards, of which I sold a certain quantity at the rate of 18*s.* 9*d.* for every 5 yards, and gained thereby as much as 180 yards cost. I demand how many yards I sold?

(19) How many pieces of Holland, each 20 ells Flem. may I have for 23*l.* 8*s.* at 6*s.* 6*d.* per ell English?

(20) How many ingots of silver, each 4 lb. 1 oz. 14 dwts. can I buy for 102*l.* 16*s.* 6*d.* supposing I give at the rate of 5*s.* 10*d.* per ounce?

(21) Suppose I give at the rate of 8¾*d.* per oz. for coffee, how many parcels, each 1 cwt. can I have for 426*l.* 16*s.*?

(22) A gentleman having an estate of 488*l.* 5*s.* per ann. is desirous to know how much he may spend daily, so that he may lay up 100 guineas at the year's end.

(23) Suppose a gentleman has an estate of 564*l.* 12*s.* per ann. and he is rated at 3*s.* 9*d.* per pound for the land-tax. Quere, his net yearly income?

(24) Suppose I give 5*s.* 9*d.* for 1 oz. of silver, how many ingots, each weighing 2 lb. 10 oz. 12 dwts. may I have for 200*l.*?

(25) A draper bought of a merchant 6 packs of cloth; every pack had 6 parcels, each parcel contained 10 pieces, and every piece was 30 yards; he gave after the rate of 2*l.* 4*s.* 3*d.* for 3 yards. I desire to know what the 6 packs cost him ?

(26) If 16 weeks' pay comes to 14*l.* 16*s.* what is that per year?

(27) A butcher goes with 116*l.* 14*s.* to Smithfield market, and buys cattle at the following prices, viz. oxen at 10*l.* each, cows at 7*l.* each, calves at 1*l.* 10*s.* each, sheep at 19*s.* each, and of each the same number. How many of each sort will the said 116*l.* 14*s.* buy?

(28) A person failing in trade owed me 560*l.* for which I received only 374*l.* 10*s.* 6½*d.* At what rate did I receive per pound?

(29) The net proceeds of a hhd. of Barbadoes sugar, weight

18 cwt. 3 qrs. 17 lb. 7 oz. was 4*l.* 14*s.* 6*d.* the custom and fees, 2*l.* 8*s.* 6*d.* Freight, 1*l.* 2*s.* 8*d.* Factorage, 4*s.* 6*d.* Pray how must I sell it per cwt. so that I may gain 5*l.* 10*s.* by the whole?

(30) A certain tower projected upon level ground a shadow to the distance of 63 yards 1 foot, when a staff 3 feet in length, perpendicularly erected, cast a shadow of 6 feet 4 inches. From hence the height of the tower is required.

(31) Suppose a person travel 285 miles in 6 days 4 hours, at what rate is that per hour, allowing 12 hours to the day?

(32) Suppose I gave 44*l.* 2*s.* for one pipe of wine, at what rate did I give per pint?

(33) The globe of the earth, under the equinoctial line, is 360 degrees in circumference; and this body being turned on its own axis, in the sidereal day, or 23 hours 56 minutes, at what rate an hour are the inhabitants of Bencoolen, situated in the midst of the torrid zone, carried from west to east by this rotation?

(34) What is the value of one grain of gold, when one ounce costs 5 guineas?

(35) If 12 apples are worth 21 pears, and 3 pears cost a halfpenny, what will be the price of fourscore and four apples?

(36) It is a rule in some parishes to assess the inhabitants in proportion to eight-tenths of their rents. What is the yearly rent of that house which pays 8*l.* 10*s.* to the king under this limitation, at 4*s.* in the pound?

(37) If 19 yards of yard-wide stuff exactly line 14 yards of silk of another breadth, how many yards of the latter will line 184 pieces of the former, each piece holding 28½ yards?

(38) In 117 times 406 pieces of coin, worth 3*s.* 8*d.* a piece, how many rees at 20 for 3*d.* English?

(39) A merchant bought 274 ells Flemish of Holland, for 4*s.* per ell, and sold it again for 7*s.* 10*d.* per ell English. What did he gain by the whole?

(40) A May-pole 50 feet 11 inches long, when the sun is on the meridian, will cast a shadow 98 feet 6 inches long; I would thereby find the breadth of a river, that runs due E. and W. within 20 feet 6 inches on the north side of the foot of a steeple, 300 feet 8 inches high, which at the same time throws the

extremity of its shadow 30 feet 9 inches beyond the stream.

(41) If 2 men in three days will earn 15*s.* how much will seven men earn in the same time?

(42) How far will one be able to travel in 9 days 8 hours, at the rate of 12 miles every four hours, allowing 12 hours to a travelling day?

(43) What will 1000 yards of walling amount to, at the rate of 4*s.* 6*d.* per rod?

(44) A factor bought 64 pieces of Holland, which cost him 352*l.* at 5*s.* 6*d.* per ell Flemish. I demand how many yards there were in all, and how many ells English in each piece.

(45) If 100*l.* in 12 months gain 4*l.* 15*s.* what will 40*l.* 10*s.* gain in the same time, the interest being at the same rate?

(46) If 12 yards of yard-wide stuff exactly line 8 yards of silk of another breadth, how many yards of the latter will line 24 pieces of the former, each piece containing 20 yards?

(47) What is the quarter's rent of 240 acres of land, at 1*l.* 19*s.* 6*d.* per acre per annum?

(48) A person owes 1000*l.* but not being able to pay the whole, compounds with his creditors to pay half-a-guinea in the pound. How much does he pay his creditors?

(49) When the sun is in the meridian at Soho-square, in what time will it be so at Tyburn, lying due west of it at the distance of a measured mile, in the latitude of 51½ degrees north, where a degree of longitude measures 37 miles, 2 furlongs, 37 poles, 5 feet, and 6 inches, known by the diurnal rotation of the earth to pass in 4 minutes' time?

(50) How many pieces of Holland, each 33 ells Flem. 1 qr. 2 na. can I have for 118*l.* 17*s.* 7½*d.* when 4 ells English cost 1*l.* 7*s.* 10*d.*?

(51) I laid out 100*l.* upon serges and shalloons: the total value of the shalloons was 60*l.* and the total quantity of serge 236 yards; also, for every two yards of serge, I had three of shalloon. How much shalloon was there, and what was the value of one yard of each sort?

(52) If 14*s.* will buy 8 lb. of tobacco, how much will 4*l.* 19*s.* 1½*d.* buy at the same rate?

(53) What will the carriage of 20 cwt. 2 qrs. 16 lb. come to at 7*s.* per cwt. ?

(54) Bought a pipe of Port wine, for which I gave 25*l.* 4*s.* but it leaked out 12 gallons; the remainder I sold at the rate of 18*d.* per quart. What was my gain or loss in the whole ?

(55) If 7*l.* 4*s.* 9*d.* be paid for the carriage of 20 cwt. 2 qrs. 16 lb. at what rate is that per pound ?

(56) How many bricks, 9 inches long, and 4 inches wide, will floor a room that is 20 feet square ?

QUESTIONS *for Exercise at leisure Hours.*

(57) I am dispatched on a commission from London to Edinburgh, distant by computation 350 miles, and my route is settled at 22 miles a-day; you, four days after, are sent after me with fresh orders, and are to travel 32 miles a-day. Whereabout on the road shall I be overtaken by you ?

(58) In the year 1582, Pope Gregory reformed the Julian calendar, ordaining, that as the year is found to consist only of 365 days, 5 hours, and about 49 minutes, in order to prevent the inconvenience of carrying the account of time too forward, by taking the solar year at 365 days and 6 hours full, which in a series of years must bring Lady-day to Michaelmas, that the Christian states for the future should drop three days in account every 400 years: that is to say, for each of the three first centuries in that space of time, the intercalary day in February should be omitted, but retained as formerly in the last century, beginning with the year 1700, when 10 whole days were sunk at once, by which artifice the variation of time will not, at least for a long space, be very considerable; according to his regulation, it is required to know in what year of Christ, the New Style, as it is called, will be twenty days, as it is now only 11, before the Old Style, which makes no such allowance.

(59) Suppose the sea allowance for the common men to be 5lb. of beef, and 3lb. of biscuit a-day, for a mess of four people, and that the price of the former be to the king 2½*d.* per lb. and of the latter 1½*d.*: such was the ship's company, that their meat cost the government 12 guineas per day; pray what did it pay for their bread per week ?

(60) A. set out from London for Lincoln at the very same time that B. at Lincoln set forward for London, distant 135 miles; at eight hours' end they met on the road, and then it appeared that A. had rode $2\frac{1}{2}$ miles an hour more than B. At what rate an hour did each of them travel?

(61) A. can do a piece of work in 10 days, B. alone in 13. Set them both about it together, in what time will it be finished?

(62) B. and C. together can build a boat in 18 days; with the assistance of A. they can do it in 11 days. In what time would A. do it himself?

(63) In some parishes in the country they take off 3*l.* one year in 17, from the rents, in assessing the farms. What will the landlord receive net out of a farm of 140*l.* a-year in those places, when the king's tax is 4*s.* in the pound?

(64) A tradesman begins the world with 1000*l.* and finds that he can gain 1000*l.* in five years by land trade alone, and that he can gain 1000*l.* in eight years by sea trade alone; and likewise that he spends 1000*l.* in $2\frac{1}{2}$ years by gaming. How long will his estate last if he follow all three?

(65) If I leave Exeter at ten o'clock on Tuesday morning for London, and ride at the rate of 2 miles an hour without intermission; you set out of London for Exeter at six the same evening, and ride three miles an hour constantly: the question is, whereabout on the road you and I shall meet, if the distance of the two cities be 170 miles.

(66) If the Sun move every day one degree, and the Moon thirteen, and at a certain time the Sun be at the beginning of Cancer, and in three days after the Moon in the beginning of Aries, the place of their next following conjunction is required.

(67) As I was beating on the forest grounds,
Up starts a hare before my two greyhounds:
The dogs, being light of foot, did fairly run,
Unto her fifteen rods, just twenty-one.
The distance that she started up before
Was fourscore sixteen rods just, and no more:
Now this I'd have you unto me declare,
How far they ran before they caught the hare?

OF SOUND.

SOUND not interrupted is, by experiments, found uniformly to move about 1150 feet in one second of time.

(68) How long after firing the warning-gun in Hyde-Park may the same be heard at Highgate, taking the distance at $5\frac{2}{3}$ miles?

(69) If I see the flash of a piece of ordnance, fired by a vessel in distress at sea, which happens, we will suppose, nearly at the instant of its going off, and hear the report a minute and three seconds afterwards, how far is she off, reckoning for the passage of sound as before?

OF THE LEVERS—

There being three orders of levers, or three varieties wherein the weight, prop, or moving power, may be differently applied to the vectis, or inflexible bar, in order to effect mechanical operations in a convenient manner.

For the First Order, see page 71.

Of the Second and Third Order of LEVERS.

In mechanics, a lever of the second order is, where the power acts at one end, the prop fixed directly at the other, and the weight somewhere between them.

In this order of levers their force is in a contra proportion to their length.

In a lever of the third order the prop is planted at one end of the bar, the weight at the other end, and the moving force somewhere between.

(70) If a lever be 100 inches long, what weight lying $7\frac{1}{2}$ inches from the end resting on a pavement, may be moved with the force of 168 lb. lifting at the other end of the lever?

(71) A water-wheel turns a crank, working three pump-rods, fixed just six feet from the joint or pin; by which their several levers, each nine feet in length, are fastened, for the sake of the intended motion, at one end. The suckers of the pumps being worked

by the other, shows them to be levers of the third order. Now I would know what the length of the stroke in each of the barrels will be, if the crank be made to play just nine inches round its centre.

(72) With what force ought that water-wheel to be driven, which, circumstanced as in the last question, raises 3 cubic feet of water at every revolution of the wheel, each experimentally weighing $62\frac{1}{2}$ lb. Avoirdupois, the friction of the machine rejected?

Motion of Bodies, with their Velocities.

1. If the quantities of matter, in any two or more bodies put in motion, be equal, the forces wherewith they are moved will be in proportion to their velocities.
2. If the velocities of these bodies be equal, their forces will be directly as the quantities of matter contained in them.
3. If both the quantities of matter and the velocities be unequal, the force with which the bodies are moved will be in a proportion compounded of the quantities of matter they contain, and of the velocities wherewith they move.

(73) There are two bodies; the one contains 25 times the matter of the other (or is 25 times heavier), but the lesser moves with 1000 times the swiftness of the greater. In what proportion are the forces by which they are moved?

(74) There are two bodies; one of which weighs 100 lb. the other 60 lb. but the lesser body is impelled by a force 8 times greater than the other. The proportion of the velocities wherewith these bodies move is required.

(75) There are two bodies; the greater contains 8 times the quantity of matter of the less, and is moved with a force 48 times greater. The ratio of the velocity of these two bodies is required.

1. In comparing the motions of bodies, if their velocities be equal, the spaces described by them are in direct proportion of the times in which they are described.
2. If the times be equal, then the spaces described will be as their velocities.
3. If the times and the velocities be unequal, the spaces

will be in a proportion compounded of the times and velocities.

(76) There are two bodies, one of which moves forty times swifter than the other, but the swifter body has moved but one minute, whereas the other has been in motion two hours. The ratio of the spaces described by these two bodies is required.

(77) Suppose one body to move thirty times swifter than another; as also the swifter to move 12 minutes, the other only 1: what difference will there be between the spaces by them described, supposing the last has moved 60 inches?

(78) There are two bodies; one whereof has described fifty miles, the other only five; but the first has moved with five times the velocity of the second. What is the ratio then of the times they have been describing those spaces?

XIII. RECIPROCAL PROPORTION,

OR,

The RULE OF THREE INVERSE.

Reciprocal Proportion is, when of four numbers the third bears the same ratio to the first as the second to the fourth; therefore the less the third term is in respect to the first, the greater will the fourth term be in respect to the second.

RULE.

Multiply the first and second terms together, and divide their product by the third term: the quotient will be the answer.

EXAMPLES.

(1) If 48 men can perform a piece of work in 12 days, how many men can do the same in 72 days?

(2) How much in length, that is four inches broad, will make a foot square?

(3) Suppose I lend my friend 500*l.* for six months, allowing the month to be 30 days, afterwards he would requite my kindness by lending me 220*l.* Required

the time I must have it, to requite my former kindness.

(4) A garrison being besieged, has three months' provisions in it, at the rate of 14 ounces per day each man: but being informed that it cannot be relieved till the end of eight months, how many ounces per day must each man have, that the said provisions may last that time?

(5) If when the price of a bushel of wheat is 4*s.* 6*d.* the penny-loaf weighs 12 oz., what must the penny-loaf weigh when the said bushel is worth but 3*s.*?

(6) Suppose 275 yards of cloth, which is 5 qrs. wide, make coats for 130 men; how many yards of shalloon, of 3 qrs. wide, will line the said coats?

(7) In what time will 600*l.* gain 50*l.* interest, when 80*l.* does it in 15 years?

(8) Suppose 200*l.* would defray the expenses of ten men for 43 weeks and 5 days, how long would six men be in spending the same sum:

(9) How many yards of paper, that is 3 qrs. wide, will hang a room that is 30 feet long, and 24 feet wide?

(10) A garrison consisting of 1500 men, being besieged, have provisions only for three months: but it being necessary they should hold out five months, how many men must depart that the said provisions may serve that time?

(11) There is a vessel, having a cock, which will empty it in six hours. I demand how many cocks of the same capacity there must be to empty the said vessel in fifteen minutes.

A LEVER of the First Order.

A lever of the first order has the power at one of its ends; the weight to be raised is put at the other, and the fulcrum or prop somewhere between them.

In this order, the power applied at one end will be reciprocally proportional to the distances of those ends from the fulcrum, or point supported; or in the steel-yards, as the distance of the weight from the point of suspension.

QUESTIONS *for Exercise at leisure Hours.*

(12) What weight will a person be able to raise, who presses

with the force of a hundred and half on the end of an equipoised handspike 100 inches long, which is to meet with a convenient prop exactly $7\frac{1}{2}$ inches above the other end of the machine?

(13) What weight, hung at 70 inches' distance from the fulcrum of a steel-yard, will equipoise a hhd. of tobacco, weighing $9\frac{1}{2}$ cwt. freely suspended at 2 inches' distance on the contrary side?

Motion of Bodies, with their Velocities.

In comparing the motion of bodies, the ratio or proportion between their velocities will be compounded of the direct ratio of the forces wherewith they are moved, and the reciprocal of the quantities of matter they contain.

(14) The battering ram of Vespasian weighed, suppose 100000lb. and was moved, let us admit, with such a velocity, by strength of hands, as to pass through 20 feet in one second of time; and this was found sufficient to demolish the walls of Jerusalem. With what velocity must a cannon ball that weighs but 30lb. be moved in order to do the same execution?

(15) A body weighing 200lb. is impelled by such a force, as to send it 100 feet in a second. With what velocity would a body of 8lb. move, if it were impelled by the same force?

XIV. CONTRACTIONS in the RULE of THREE.

This being considered, you may oftentimes perform the work much shorter than by the general rule.

1. Divide the third term by the first, multiply that quotient by the second, and their product will be the answer.

EXAMPLES.

(1) If four yards of broad-cloth cost 3*l.* 17*s.* 6*d.* what will a piece, containing 28 yards, come to, at the same rate?

(2) If three chests of tea, each 3 cwt., cost 11*l.* 13*s.* what must I give for 72 cwt. of the same?

2. Divide the second term by the first, multiply that quotient by the third, and their product will be the answer.

EXAMPLES.

(3) Suppose I give 78*l.* for 26 cwt. of cheese, what must be given for 156 cwt. of the same?

(4) If for 3lb. of tea I give 18*s.* what is the value of 1 cwt. of the same?

3. Divide the first term by the second, and divide the third by that quotient; which will give the answer.

EXAMPLES.

(5) Suppose I give for 6 gowns, each containing 8 yards of stuff, 6*l.* what must be given for 64 yards at the same rate?

(6) If I give 12*s.* for 24 lb. of sugar, what must be given for $1\frac{1}{2}$ cwt. of the same?

4. Divide the first term by the third, divide the second by that quotient, and the last quotient will be the answer.

EXAMPLES.

(7) Suppose I give for 3 pieces of broad cloth, each 24 yards, 64*l.* 14*s.* what must be given for 8 yards of the same?

(8) If for 6 parcels of tea, each 3 lb. I give 12*l.* 19*s.* what must I give for 6 lb. of the same at that rate?

5. Divide the first term by the third, multiply the second by that quotient, and the product will be the answer.

EXAMPLES.

(9) How much in length, that is 3 inches broad, will make a foot square?

(10) If for 48*s.* I have 225 cwt. carried 512 miles, how many hundred can I have carried 64 miles for the same money?

XV. COMPOUND PROPORTION,

OR

The RULE of FIVE,

IS so called, from its having five numbers or terms given to find a sixth. Here, if the proportion be direct, the sixth term must bear such a proportion to the fourth and fifth as the third bears to the first and second. But if the proportion be inverse, then the sixth term must bear such proportion to the fourth and fifth as the first bears to the second and third, or as the second bears to the first and third.

The three first terms are a supposition, the two last a demand.

RULES.

1. Let the principal cause of gain, loss, or action, &c. be put in the first place.
2. Let that which denotes time, distance of place, &c. be in the second place, and the remaining one in the third place.
3. Place the other two terms which move the question underneath those of the same name.
4. If the blank or term sought fall under the third term, multiply the first two terms together for a divisor, and the last three for a dividend: the quotient arising from them will be the answer, or sixth term.
5. If the blank fall under the first or second term, multiply the third and fourth terms together for a divisor, and the other three for a dividend: the quotient arising from them will be the answer.

PROOF.

By two Statings.

EXAMPLES.

(1) If 6 men can mow 72 acres of grass in 12 days, how many men can mow 120 acres in 4 days?

(2) Suppose 2 bushels of wheat will be sufficient for a family of six persons 12 days, how many bushels will serve 36 persons 4 days?

(3) Suppose the salary of 6 persons for 21 weeks is 120*l.* what will be the salary of 14 persons for 46 weeks?

(4) If, for the carriage of 40 cwt. 100 miles, I give 9*l.* 5*s.* what weight can I have carried 125 miles for 70*l.* 10*s.* 3¼*d.* at the same rate?

(5) An usurer put out 120*l.* to receive interest for the same: but when it had continued 9 months, he took it up, received for the principal and interest 125*l.* 8*s.* I demand at what rate per cent. per annum he received?

(6) What is the interest of 259*l.* 13*s.* 5*d.* for 20 weeks, at 5*l.* per cent. per annum?

(7) If a quantity of provisions serve 1400 men 20 weeks, at the rate of 14 ounces per day each man, how many men will the same provisions maintain for 8 months, at the rate of 8 ounces per day each?

(8) Suppose 8 men earn 5*l.* in 5 days, how many men will earn 10 guineas in 12 days?

(9) Suppose 140*l.* would defray the expenses of five men for twenty-four weeks and four days, how long would twelve men be in spending 200*l.* at the same rate?

(10) What money, at 3½*l.* per cent. per annum, will clear 38*l.* 10*s* in a year and a quarter's time?

(11) If a sack of coals be the allowance of 7 poor people for a week, how many poor belonged to that parish, which, when coals were 36*s.* per chaldron, had 41*l.* to pay in 6 weeks on that account?

QUESTIONS *for Exercise at leisure Hours.*

(12) A. and B. are on opposite sides of a wood, 135 toises or fathoms about. They begin to go round it both the same way at the same instant of time; A. goes 11 toises in 2 minutes, and B. 17 in 3. The question is, how many times will they surround this wood before the nimbler overtakes the slower?

(13) If a lever, 40 effective inches long, will, by a certain power thrown successively thereon, in 13 hours raise a weight 104 feet, in what time will two other levers, each 18 effective inches long, raise an equal weight

73 feet; the force of straight levers being in direct proportion of their lengths?

(14) A weight of $1\frac{1}{2}$lb. laid on the shoulder of a man, is no greater burden to him than its absolute weight, or 24 ounces. What difference will he feel between the said weight applied near his elbow, at 12 inches from the shoulder, and in the palm of his hand 28 inches therefrom? and how much more must his muscles then draw to support it at right angles; that is, having his arm extended right out?

(15) In giving directions for making an Italian chair, the shafts whereof were settled at 11 feet between the axle-tree, whereon is the principal bearing, and the backband, by means of which the weight is partly thrown upon the horse; a dispute arose whereabout on the shafts the centre of the body of this machine should be fixed. The coachmaker advised this to be done at 30 inches from the axle; others were of opinion, that at 24 it would be a sufficient encumbrance to the horse. Now, admitting the two passengers, with their baggage, ordinarily to weigh 2 cwt. a piece, and the body of the vehicle to be about 70lb. more; what will the beast, in both those cases, be made to bear more than his harness?

(16) Suppose a person to travel 152 miles in 7 days, when the days are 12 hours long, how many days will he be in travelling 576 miles, when the days are 16 hours long?

(17) A water-tub holds 147 gallons; the pipe usually brings in 14 gallons in 9 minutes; the tap discharges, at a medium, 40 gallons in 31 minutes. Supposing these both to be carelessly left open, and the water to be turned at two in the morning; the servant at five, finding the water running, shuts the tap, and is solicitous in what time the tub will be filled after this accident, in case the water continue flowing from the main?

(18) If the scavenger's rate, at $1\frac{1}{2}d.$ in the pound, amount to $6s.$ $7\frac{1}{2}d.$ where they ordinarily assess $\frac{4}{5}$ of the rent, what will the king's tax for that house be, at $4s.$ in the pound, rated at the full rent?

(19) If, when port wine is 17 guineas the hogshead, a company of 45 people will spend $20l.$ therein, in a certain time; what is wine a pipe, when 13 persons more will

spend 63*l.* in twice the time, drinking with equal moderation?

(20) There is an island 73 miles round, and three men start together, to travel the same way about it. A. travels 5 miles a day, B. 8, and C. 10. When will they all come together again?

(21) A certain man hired a labourer on this condition,—that for every day he worked he should receive 1*s.* but for every day he was idle he should be fined 8*d.* When 390 days were past, neither of them was indebted to the other. How many days did he work, and how many days was he idle?

(22) A. lent his friend B. fourscore and eleven guineas from the 11th of December to the 10th of May following; B. on another occasion, let A. have 100 marks, from September the 3d to Christmas following. Quere, How long ought the person obliged to let his friend use 40*l.* fully to retaliate the favour?

(23) A man hired a labourer for 40 days, on condition that he should have 20*d.* for every day he worked, and forfeit 10*d.* for every day he idled; at last he received 2*l.* 1*s.* 8*d.* for his labour. How many days did he work, and how many was he idle?

XVI. PRACTICE,

SO called from the general use it is of to all persons concerned in trade and business.

All questions in the Rule of Three, where the first term is unity or one, may be performed by this rule;

Which is by taking aliquot or even parts, by which means many tedious reductions may be avoided.

But as there are a great variety of such parts, so many, therefore, are the ways of applying them, that it would be an endless task to give all the easy methods of operation adapted to particular cases; so I shall only give the general rules, with a sufficient number of examples to illustrate them.

In order to perform expeditiously, it will be necessary that the learner get by heart the following

TABLE.

Of a Pound	*Of a Sh.*	*Of a Ton.*	*Of a Hun.*	*Of a Qr. of a C.*
s. d.	*d.*	*cwt.*	*qr. lb.*	*lb.*
1. 8 = $\frac{1}{12}$	1 = $\frac{1}{12}$	2 = $\frac{1}{10}$	1 or 28 = $\frac{1}{4}$	$3\frac{1}{2}$ = $\frac{1}{8}$
2 0 = $\frac{1}{10}$	$1\frac{1}{2}$ = $\frac{1}{8}$	$2\frac{1}{2}$ = $\frac{1}{8}$	2 or 56 = $\frac{1}{2}$	4 = $\frac{1}{7}$
2 6 = $\frac{1}{8}$	2 = $\frac{1}{6}$	4 = $\frac{1}{5}$	0 16 = $\frac{1}{7}$	7 = $\frac{1}{4}$
3 4 = $\frac{1}{6}$	3 = $\frac{1}{4}$	5 = $\frac{1}{4}$	0 14 = $\frac{1}{8}$	14 = $\frac{1}{2}$
4 0 = $\frac{1}{5}$	4 = $\frac{1}{3}$	10 = $\frac{1}{2}$		
5 0 = $\frac{1}{4}$	6 = $\frac{1}{2}$			
6 8 = $\frac{1}{3}$				
10 0 = $\frac{1}{2}$				

Case 1. When the price is less than a penny.

RULE.

Divide by the aliquot parts that are in a penny, then by 12 and 20, which will give the answer.

EXAMPLES.

(1) 2107 at $\frac{1}{4}$*d.* (2) 1470 at $\frac{1}{2}$*d.* (3) 1276 yds. at $\frac{3}{4}$*d.* per yd.

Case 2. When the price is less than a shilling.

RULE.

Take the aliquot part or parts that are in a shilling, add them together, and the sum will be the answer in shillings, &c. which, divided by 20, as before, will give pounds, &c.

EXAMPLES.

(4) 1762 at 1*d.* (5) 1400 at $1\frac{1}{4}$*d.* (6) 2462 at $1\frac{1}{2}$*d.*

(7) 1041 at $1\frac{3}{4}$*d.* (8) 2420 at 2*d.* (9) 2408 at $2\frac{1}{4}$*d.*

(10) 640 at $2\frac{1}{2}$*d.* (11) 1740 at $2\frac{3}{4}$*d.* (12) 746 at 3*d.*

(13) 1417 at $3\frac{1}{4}$*d.* (14) 3091 at $3\frac{1}{2}$*d.* (15) 214 at $3\frac{3}{4}$*d.*

(16) 2000 at 4*d.* (17) 569 at $4\frac{1}{4}$*d.* (18) 1246 at $4\frac{1}{2}$*d.*

(19) 1416 at $4\frac{3}{4}$*d.* (20) 2740 at 5*d.* (21) 2147 at $5\frac{1}{4}$*d.*

(22) 674 at $5\frac{1}{2}$*d.* (23) 1746 at $5\frac{3}{4}$*d.* (24) 1741 at 6*d.*

(25) 2142 at $6\frac{1}{4}$*d.* (26) 1040 at $6\frac{1}{2}$*d.* (27) 1746 at $6\frac{3}{4}$*d.*

(28) 1000 at 7*d.* (29) 1656 at $7\frac{1}{4}$*d.* (30) 1420 at $7\frac{1}{2}$*d.*

(31) 674 at $7\frac{3}{4}$*d.* (32) 2170 at 8*d.* (33) 1700 at $8\frac{1}{4}$*d.*

(34) 1765 at $8\frac{1}{2}$*d.* (35) 749 at $8\frac{3}{4}$*d.* (36) 1417 at 9*d.*

(37) 2373 at $9\frac{1}{4}$*d.* (38) 1476 at $9\frac{1}{2}$*d.* (39) 1760 at $9\frac{3}{4}$*d.*

(40) 6000 at 10*d.* (41) 4652 at $10\frac{1}{4}$*d.* (42) 2476 at $10\frac{1}{2}$*d.*

(43) 2176 at $10\frac{3}{4}$*d.* (44) 1276 at 11*d.* (45) 2142 at $11\frac{1}{4}$*d.*

(46) 4760 at $11\frac{1}{2}$*d.* (47) 640 at $11\frac{3}{4}$*d.*

Case 3. When the price is more than a shilling, but less than two.

RULE.

Take the part or parts, with so much of the given price as is more than a shilling (as in the last rule), which add to the given quantity, and the sum will be the answer in shillings, &c. which, divided by 20, will give pounds.

Note.—The method of performing this case being so little different from the last, I shall only give a few examples, which, by proper instructions from the tutor, will suffice.

EXAMPLES.

(48) 1074 at 1*s.* $10\frac{1}{4}$*d.* (49) 2140 at 1*s.* $0\frac{1}{4}$*d.*

(50) 1749 at 1*s.* $11\frac{1}{2}$*d.* (51) 2140 at 1*s.* 5*d.*

(52) 1453 at 1*s.* 7½*d.* (53) 1614 at 1*s.* 10*d.*

(54) 2647 at 1*s.* 11¾*d.*

Case 4. When the price consists of any even number of shillings under 20.

RULE.

Multiply the given quantity by half the price, doubling the first figure of the product for shillings; and the rest of the product will be pounds.

EXAMPLES.

(55) 2476 at 2*s.* (56) 1476 at 4*s.* (57) 276 at 6*s.*

(58) 2100 at 8*s.* (59) 274 at 10*s.* (60) 674 at 12*s.*

(61) 2680 at 14*s.* (62) 267 at 16*s.* (63) 1267 at 18*s.*

Case 5. When the price is any odd number of shillings under 20.

RULE.

Multiply the given quantity by the price; and the product will be the answer in shillings, which, divided by 20, will give pounds.

EXAMPLES.

(64) 2174 at 7*s.* (65) 1427 at 9*s.* (66) 647 at 11*s.*

(67) 267 at 13*s.* (68) 274 at 17*s.* (69) 1260 at 19*s.*

Case 6. When the price is shillings, or shillings and pence, and they an aliquot part of a pound.

RULE.

Divide by the aliquot part, and the quotient will be the answer.

EXAMPLES.

(70) 2420 at 4*s.* (71) 1764 at 5*s.* (72) 4762 at 1*s.* 8*d.*

(73) 467 at 2*s.* 6*d.* (74) 1760 at 3*s.* 4*d.* (75) 176 at 6*s.* 8*d.*

Case 7. When the price is shillings and pence, and the shillings and pence are not an aliquot part of a pound.

RULE.

Multiply the given quantity by the shillings, and take parts for the pence, &c. add them together, and the sum will be the answer in shillings, which, divided by 20, will give pounds.

EXAMPLES.

(76) 1420 at 3*s.* 3*d.* (77) 427 at 5*s.* 9*d.*

(78) 402 at 10*s.* 8¾*d.* (79) 174 at 17*s.* 9½*d.*

(80) 273 at 19*s.* 4¾*d.* (81) 260 at 14*s.* 11¾*d.*

Case 8. When the price is pounds only.

RULE.

Multiply the given quantity by the price, and the product will be the answer.

EXAMPLES.

(82) 120 at 4*l.* (83) 96 at 17*l.*

(84) 100 at 3*l.* (85) 142 at 2*l.*

Case 9. When the given price is pounds and shillings.

RULE.

Multiply the quantity given by the pounds, as in the last case; and proceed with the shillings, if they are even, as in

Case 4; but if odd, take aliquot parts, add them together, and the sum will be the answer; or reduce the given price to shillings, by which multiply the fixed quantity, and divide by 20; and this will give the answer.

EXAMPLES.

(86) 649 at 2*l.* 6*s.* (87) 526 at 7*l.* 16*s.* (88) 142 at 1*l.* 17*s.*

(89) 164 at 24*l.* 19*s.* (90) 271 at 5*l.* 7*s.* (91) 604 at 20*l.* 9*s.*

(92) 914 at 10*l.* 15*s.* (93) 737 at 1*l.* 14*s.*

Case 10. When the price is pounds, shillings, and pence, and the shillings and pence are an aliquot part of a pound.

RULE.

Multiply the given quantity by the pounds, as in the last rule; and take parts for the shillings and pence, as in Case 6; add them together, and the sum will be the answer.

EXAMPLES.

(94) 274 at 7*l.* 6*s.* 8*d.* (95) 120 at 12*l.* 3*s.* 4*d.*

(96) 97 at 9*l.* 1*s.* 8*d.* (97) 512 at 42*l.* 5*s.*

Case 11. When the price is pounds, shillings, pence, and farthings, and the shillings and pence are not an aliquot part of a pound.

RULE.

Reduce the pounds and shillings into shillings; multiply the given quantity by the shillings, as in Case 9; take parts for the pence and farthings, as in Case 2.

Note.—When the given quantity does not exceed 100, proceed as in Sect. IX.

EXAMPLES.

(98) 1472 at 4*l.* 6*s.* 7½*d.* (99) 279 at 6*l.* 11*s.* 9¾*d.*

(100) 1420 at 19*l.* 14*s.* 11¾*d.*

(101) 2074 at 1*l.* 17*s.* 5½*d.*

(102) 27 at 4*l.* 11*s.* 8¾*d.*

(103) 64 at 12*l.* 13*s.* 7¾*d.*

Case 12. When the price and quantity given are of several denominations.

RULE.

Multiply the price of one by the quantity given, and take parts for quarters, pounds, &c. Add them together, and the sum will be the answer.

EXAMPLES.

(104) Bought 7 cwt. 3 qrs. 18 lb. of sugar, at 17*s.* 6*d.* per cwt. What comes it to?

(105) Sold 420 oz. 15 dwts. 16 grs. of gold, at 3*l.* 16*s.* 10½*d.* per ounce. What comes it to?

(106) Bought tobacco, at 3*l.* 17*s* 4½*d.* per cwt. What is the worth of 72 cwt. 3 qrs. 19 lb.?

(107) Bought 12 cwt. 1 qr. 17 lb. of hops, at 4*l.* 4*s.* 4*d.* per cwt. What do they stand me in?

(108) Sold 23 cwt. 18 lb. of sugar, at 4*l.* 14*s.* 1½*d.* per cwt. What comes it to?

(109) What is the rent of 476 acres, 3 roods, 28 perches, at 3*l.* 7*s.* 11*d.* per acre?

(110) Sold 16 cwt. 2 qrs. of tallow, at 2*l.* 6*s.* 11*d.* per cwt. What comes it to?

(111) Sold 48 cwt. 2 qrs. 7 lb. of fine hyson tea, at 74*l.* 16*s.* 6*d.* per cwt. What must I receive for the same?

(112) What is the value of 24 lb. of double-refined sugar, at 4*l.* 17*s.* per cwt.?

(113) What is the value of 17 lb. of Malaga raisins, at 3*l.* 5*s.* 4*d.* per cwt.?

Note. When the pupil is perfect in all the rules to the preceding Cases, he may then be taught the contractions.

EXAMPLES.

(1) 1276 at ¾*d.*

(2) 1740 at 2¾*d.*

(3) 1420 at 7¼*d.*

(4) 3162 at 1*s.* 7*d.*

(5) 427 at 5*s.* 9*d.*

(6) 246 at 17*s* 10*d.*

(7) 241 at 6*l.* 19*s.*

(8) 641 at 1*l.* 15*s.*

XVII. TARE and TRETT.

In this rule there are six things to be observed, viz.
1. The Gross Weight. 2. Tare. 3. Trett. 4. Suttle. 5. Cloff. And, 6. The Net Weight.

1. The Gross Weight is the whole weight of the goods, and that which they are packed up in.
2. Tare is an allowance made to the purchaser for the weight of the box, bag, barrel, or whatever contains the goods bought, and is either
   At so much in the whole gross weight;
   At so much per box, bag, &c.—or,
   At so much per cent.
3. Trett is an abatement of 4 lb. per 104 lb. and is the twenty-sixth part allowed for waste, dust, &c. made by the merchant to the buyer.
4. Suttle is when the tare is deducted from the gross.
5. Cloff is an allowance of 2 lb. to the citizens of London on every draught above 3 cwt. on some sorts of goods, as beaver, galls, madder, argol, &c.
6. Net weight is when all allowances are deducted from the gross.

Case 1. When the tare is at so much in the gross weight, to find the net.

RULE.

Subtract the tare from the gross, and the remainder is the net weight.

EXAMPLES.

(1) What is the net weight of 24 hogsheads of tobacco, each weighing 6 cwt. 2 qrs. 17 lb. gross? Tare in the whole 17 cwt. 3 qrs. 27 lb.

(2) What is the net weight of 5 hhds. of sugar, weighing as follows, viz.

	cwt.	*qrs.*	*lb.*		*qrs.*	*lb.*
No. 1.	4	2	14	Tare	0	21
2.	3	0	17		0	18
3.	5	3	10		1	11

	cwt.	qrs.	lb.		qrs.	lb.
No. 4.	6.	1	16	Tare	0	27
5.	3	2	18		0	19
Gross						
Tare						
Net						

Case 2. When tare is at so much per box, bag, &c. to find the net weight.

RULE.

Multiply the tare of each box, bag, &c. by the number of boxes, &c. the product subtract from the gross as before, and the remainder will be the net weight.

EXAMPLES.

(3) What is the net weight of 8 frails of raisins, each weighing 2 cwt. 3 qrs. gross, tare at 22 lb. per frail?

(4) What is the net weight of 40 bales of silk, each weighing 3 cwt. 27 lb. gross, tare at 18 lb. per bale?

Case 3. When tare is at so much per cwt. to find the net weight.

RULE.

Divide the whole gross by the said part or parts that the tare is of a cwt. and the quotient thence arising will be the tare, which, subtracted from the gross, as before, will give the net weight.

EXAMPLES.

(5) What is the net weight of 22 barrels of figs, each 2 cwt. 1 qr. 17 lb. gross, tare at 14 lb. per cwt.?

(6) What is the net weight of 9 hhds. of sugar, each weighing 6 cwt. 2 qrs. 12 lb. gross, tare at 17lb. per cwt.?

Case 4. When trett is allowed with the tare, to find the net weight.

RULE.

Find the tare as before, and subtract it from the gross; the remainder will be the suttle, which divide by 26; and the quotient will be the trett, which subtract from the suttle; the remainder will be the net weight.

EXAMPLES.

(7) What is the net weight of 16 cwt. 3 qrs. gross; tare 1 cwt. 1 qr. 12 lb. and trett 4 lb. per 104 lb.?

(8) In 27 bags of coffee, each weighing, gross, 2 cwt. 3 qrs. 27 lb. tare 13 lb. per 112 lb.? and trett 4 lb. per 104 lb. what is the net weight?

Case 5. When cloff is allowed, to find the net weight.

RULE.

Divide the whole gross by 168 (2 lb. being 168th part of 3 cwt. or 336 lb.); or you may divide the number of cwts. by 3, which brings them into 3 cwts. Then 2 lb. being allowed for every 3 cwt. so as many 3 cwts. as it produces, so many 2 lbs. it will allow, which divide by 56 (the double pounds in a cwt.), the quotient will be the hundreds, and the remainder will be so many 2 lbs., to which adding what may be allowed for the odd cwts. qrs. and lbs. of the given weight, will make the whole cloff, which subtract from the gross, and the remainder will be the net weight.

EXAMPLES.

(9) What will the net weight of 5647 cwt. 3 qrs. 13 lb. gross, allowing for cloff 2 lb. for every 3 cwt.?

(10) What is the net weight of 14 barrels of spices, weighing altogether 42 cwt. 3 qrs. gross, allowing for cloff 2 lb. per 3 cwt.?

Case 6. When tare, trett, and cloff, are allowed with any quantity (gross), to find the net weight.

RULE.

For the tare and trett, proceed as in Case 4, and the remainder, which was called the net there, will be the

suttle here, which to find the cloff of, proceed as in the last Case.

EXAMPLES.

(11) What is the net weight of 15 cwt. 3 qrs. 20 lb. gross, allowing for tare 7 lb. per cwt.; trett 4lb. per 104 lb. and cloff 2 lb. for 5 cwt.?

(12) What is the net weight of 4 hhds. of sugar, weighing as follows, viz.

	cwt.	*qrs.*	*lb.*	
No. 1.	4	2	17	Tare 8 lb. per cwt. Trett 4 lb. per 104 lb. Cloff 2 lb. per 3 cwt.
2.	5	3	14	
3.	3	2	26	
4.	6	2	12	

QUESTIONS for EXERCISE.

(13) The net proceeds of a hogshead of Barbadoes sugar, were 4*l.* 14*s.* 6*d.* the custom and fees 2*l.* 8*s.* 6*d.* Freight 22*s.* 8*d.* Factorage 4*s.* 9*d.* The gross weight was 9 cwt. 3 qrs. 10 lb. Tare 1 lb. in 10. Pray then how was the sugar rated in the bill of parcels?

(14) I have imported 80 jars of Lucca oil, each containing 1180 solid inches. What came the freight to at 4*s.* 6*d.* per cwt. Tare 1 lb. in 10, counting $7\frac{1}{2}$ lb. of oil to the wine gallon of 231 cubic inches?

XVIII. SIMPLE INTEREST

IS that which arises only from the principal, and is a profit allowed by the borrower to the lender, for the loan or forbearance of any sum of money, for some determined space of time, and at any rate (per cent. per annum) agreed upon; which, according to law, must not exceed 5*l.* for the use or interest of 100*l.* principal, called cent., for 12 months, called annum.

The amount is the principal and interest added together.

Note.—The rules for simple interest serve also for calculating factorage, brokerage, insurance, purchasing of stocks, or any thing else, that is rated at so much per cent.

Case 1. To find the interest of any sum of money, for any number of years.

RULE.

Multiply the principal by the rate per cent.; that product, divided by 100, will give the interest for a year, which multiply by the number of years given, and the product will be the answer. Or, by taking the aliquot part or parts with the given rate that are in 100*l.*

EXAMPLES.

(1) What is the interest of 824*l.* 18*s.* 2*d.* for a year, at four per cent. per annum?

(2) What is the interest of 500*l.* for 4 years, at 5 per cent. per annum?

(3) What is the amount of 264*l.* 4*d.* for 12 years at 5 per cent. per annum.

Case 2. When the rate per cent. is $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, more than the pounds given in the said rate.

RULE.

Multiply the principal by the pounds in the rate per cent.; then take parts for $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$, from the principal, which add to the product, and the sum divide by 100, as before.

EXAMPLES.

(4) What is the interest of 246*l.* 18*s.* for a year, at $4\frac{1}{4}$ per cent. per annum?

(5) What is the interest of 140*l.* 10*s.* for 7 years, at $4\frac{1}{2}$ per cent. per annum?

(6) What is the amount of 470*l.* for 5 years, at $3\frac{3}{4}$ per cent. per annum?

COMMISSION

IS an allowance from a merchant to his factor or correspondent abroad, in buying and selling of goods, and is at a certain rate per cent. according to the custom of the country where the factor resides.

This, as well as brokerage and insurance, is computed in the same manner as for even pounds, and for $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$, as in the last Case.

EXAMPLES.

(7) My factor writes me word, that he has bought goods, upon my account, to the value of 474*l.* 14*s.* 6*d.* I demand to know what his commission comes to at $3\frac{1}{4}$ per cent.

(8) Suppose I allow my correspondent $2\frac{1}{2}$ per cent. for his commission; what is his demand on the disbursement of 742*l.* 12*s.* 6*d.*?

BROKERAGE

IS an allowance or fee paid to a person called a broker, for assisting others in buying or disposing of their goods; and in the city of London they are not to act without a licence from the lord mayor.

Case 3. To find the brokerage for any sum, at any rate under 1*l.* per cent.

RULE.

Divide the given sum by 100, and it will give the interest at 1*l.* per cent. which interest you must take parts from, with the rate per cent. and add them together; the sum will be the brokerage required.

EXAMPLES.

(9) What is the brokerage of 420*l.* 12*s.* 6*d.* at 6*s.* 4*d.* per cent.?

(10) Suppose I employ a broker, who sells goods for me, to the value of 1000*l.*, what may he demand for brokerage, it being at 4*s.* 6*d.* per cent.?

(11) Suppose a broker disposes of goods for me, to the amount of 540*l.* 10*s.* what comes the brokerage to, at 13*s.* 10*d.* per cent.?

(12) What is the brokerage of 2474*l.* 15*s.* at 19*s.* $9\frac{1}{2}$*d.* per cent.?

INSURANCE

IS a contract or agreement whereby one or more persons, called insurers, &c. oblige themselves to answer for the loss or damage of ships, houses, goods, &c. by storms, fires, &c. in consideration of a premium paid by the proprietors of the thing injured.

To find the insurance of any sum at any rate under 1*l.* proceed as in the last Case; if above, as in Case 1.

EXAMPLES.

(13) Suppose I insure for 1460*l.* at 2*s.* 6*d.* per cent. per ann. what does the insurance come to?

(14) What is the insurance of 2460*l.* at 10*l.* 15*s.* per cent.?

(15) Shipped at Jamaica, goods to the value of 2500*l.* upon which I made an insurance at $6\frac{7}{8}$ per cent. What does it come to?

(16) What is the insurance of an East-India ship and cargo, valued at 7406*l.* 17*s.* 6*d.* at $15\frac{3}{4}$ per cent.?

PURCHASING OF STOCKS.

STOCKS are the public funds of the nation, the shares of which being transferrable from one person to another, occasions that extensive business called Stock-jobbing.

RULE.

Multiply the sum to be purchased, by the excess of the rate per cent. above 100; the product divide by 100, as before, and the quotient added to the given sum will give the required purchase.

If under par, i.e. if under 100 per cent. proceed as in Case 2.

EXAMPLES.

(17) What is the purchase of 400*l.* South-Sea stock, at 120*l.* 5*s.* per cent.?

(18) What is the purchase of 470*l.* bank stock, at $87\frac{3}{4}$ per cent.?

(19) What is the purchase of 2470*l.* 17*s.* 10*d.* bank annuities, at $103\frac{1}{2}$ per cent.?

(20) What is the purchase of 876*l.* India stock, at $114\frac{5}{8}$ per cent.?

Case 4. When the interest is for $\frac{3}{4}$, $\frac{1}{2}$, or $\frac{1}{4}$, of a year, or any number of years besides.

RULE.

Find the interest for the years, as in Case 1, then for $\frac{3}{4}$, $\frac{1}{2}$, or $\frac{1}{4}$, take parts from the interest of one year; i. e. for $\frac{3}{4}$, take one-fourth part of the said interest; for $\frac{1}{2}$, take one-half; and for $\frac{1}{4}$, take the compound parts of $\frac{1}{4}$, that is, a half for $\frac{1}{2}$, then half of that for $\frac{1}{4}$, which, added to the interest for years (if any), the sum will be the interest required.

EXAMPLES.

(21) What is the interest of 427*l.* 10*s.* for four months, at 4 per cent. per annum?

(22) What is the interest of 246*l.* 12*s.* 6*d.* for $1\frac{3}{4}$ year at 5 per cent. per annum?

(23) Lent 209*l.* 18*s.* upon a mortgage, to receive interest for the same, at $4\frac{3}{4}$ per cent. per annum, till it was paid off, which was not till the end of $4\frac{1}{2}$ years after. Now I should be glad to know what is due to me.

(24) A gentleman dying left his daughter 604*l.* 17*s.* 6*d.* for her fortune, to be paid her when at age, with interest, at $5\frac{1}{2}$ per cent. per annum. Now she came of age in 3 years 9 months after her father's death. What is the amount of her fortune? that, is, What is she to receive in all, principal and interest.

Case 5. When the interest required is for any number of weeks.

RULE.

1. Find the interest of the given sum for a year, as in Case 1.

2. Say, as 52 weeks : are to that interest of the given sum :: so are the weeks given : to the interest required. Or you may divide the given weeks into aliquot parts of a year, or 52 weeks.

EXAMPLES.

(25) What is the interest of 400*l.* 15*s.* for 4 weeks at 4 per cent. per annum?

(26) What is the amount of 600*l.* for 26 weeks, at $3\frac{3}{4}$ per cent. per annum?

(27) What is the interest of 740*l.* for 4 years and 42 weeks, at 5 per cent. per annum?

(28) What is the amount of 200*l.* for 5 years and 50 weeks, at $4\frac{1}{2}$ per cent. per annum?

Case 6. To find the interest of any sum, for any number of days.

RULE.

1. Say, As 365 days : are to the interest of the given sum for a year :: so are the days given : to the interest required.

2. When convenient, divide the days into aliquot parts of a year, or 365. Or,

3. Reduce the principal into pence, which multiply by the number of days, and that product by the rate per cent. for a dividend: then multiply 365 (the days in a year) by 199 for a divisor, by which divide the dividend, and the quotient will be the answer in pence, which bring to pounds. Or,

4. Multiply the given sum by the number of days, and divide the product by 7300 (viz. $\frac{100 \times 365}{5}$) the quotient will give the interest at 5 per cent. and for a higher or lower rate, take aliquot parts for a difference, which add or subtract accordingly.

A TABLE of Days for any given time less than a Year.

Days.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.
1	1	32	60	91	121	152	182	213	244	274	305	335
2	2	33	61	92	122	153	183	214	245	275	306	336
3	3	34	62	93	123	154	184	215	246	276	307	337
4	4	35	63	94	124	155	185	216	247	277	308	338
5	5	36	64	95	125	156	186	217	248	278	309	339
6	6	37	65	96	126	157	187	218	249	279	310	340
7	7	38	66	97	127	158	188	219	250	280	311	341
8	8	39	67	98	128	159	189	220	251	281	312	342
9	9	40	68	99	129	160	190	221	252	282	313	343
10	10	41	69	100	130	161	191	222	253	283	314	344
11	11	42	70	101	131	162	192	223	254	284	315	345
12	12	43	71	102	132	163	193	224	255	285	316	346
13	13	44	72	103	133	164	194	225	256	286	317	347
14	14	45	73	104	134	165	195	226	257	287	318	348
15	15	46	74	105	135	166	196	227	258	288	319	349
16	16	47	75	106	136	167	197	228	259	289	320	350
17	17	48	76	107	137	168	198	229	260	290	321	351
18	18	49	77	108	138	169	199	230	261	291	322	352
19	19	50	78	109	139	170	200	231	262	292	323	353
20	20	51	79	110	140	171	201	232	263	293	324	354
21	21	52	80	111	141	172	202	233	264	294	325	355
22	22	53	81	112	142	173	203	234	265	295	326	356
23	23	54	82	113	143	174	204	235	266	296	327	357
24	24	55	83	114	144	175	205	236	267	297	328	358
25	25	56	84	115	145	176	206	237	268	298	329	359
26	26	57	85	116	146	177	207	238	269	299	330	360
27	27	58	86	117	147	178	208	239	270	300	331	361
28	28	59	87	118	148	179	209	240	271	301	332	362
29	29	60	88	119	149	180	210	241	272	302	333	363
30	30		89	120	150	181	211	242	273	303	334	364
31	31		90		151		212	243		304		365

The USE of the TABLE.

First, to know the number of days from the beginning of the year to any given day of any month.

This is obtained by inspection only; thus, from January the 1st, to August the 8th, are 220 days; to November the 24th, are 328 days, &c.

Secondly, To know what is the number of days from any given day of any month, to the end of the year.

Suppose August 8th, then from - - - - -	365 days,
Subtract the number answering to August 8, -	220
There remains the number of days, viz. -	145

Thirdly to find the number of days between the given day of any month, and the given day of any other month, in the same year.

For instance, To know how many days there are between May the 9th, and November the 5th.

Thus, from the number answering to Nov. 5, -	309 days,
Subtract that answering to May 9, - - - -	129
The remainder is the number of days sought, viz.	180

Fourthly, To find the number of days from any given day of any month in one year, to any given day of any month in the next year.

How many days is it from October the 12th in one year, to June the 10th in the next?

Thus, from the days of a whole year, - - -	365 days,
Subtract the number answering to Oct. 12. viz. -	285
Remains the number to the end of the year, -	80
To which add the number to June 10, - - -	161
The sum is the number of days required, viz. -	241

And thus is the number of days readily found for any interval of time given; in the same year completely; or which is part of one, and part of another year.

EXAMPLES.

(29) What is the interest of 200*l.* for 73 days, at $3\frac{1}{2}$ per cent. per annum?

(30) What is the amount of 340*l.* 10*s.* from January 1, 1789, to July 18 following, at 5 per cent. per annum?

(31) What is the interest of 500*l.* from December 4, 1791, to March 10, 1793, at 4 per cent. per annum?

Case 7. When the amount, time, and rate per cent. are given, to find the principal.

RULE.

1. Say, As the amount of 100*l.* at the rate and time given : is to 100*l.* :: so is the amount given : to the principal required. Or,

2. The examples in this and the two following Cases may be solved by the rule in Sect. XV.

EXAMPLES.

(32) What principal, being put to interest for $9\frac{1}{2}$ years, at $4\frac{1}{2}$ per cent. per annum, will amount to 856*l.* 10*s.*?

(33) What principal, being put out to interest for $7\frac{3}{4}$ years, will amount to 614*l.* 3*s.* 11*d.* at 3 per cent. per annum?

Case 8. When the principal, rate per cent., and the amount are given, to find the time.

RULE.

Say, As the interest of the principal for a year : is to 1 year :: so is the whole interest : to the time required.

EXAMPLES.

(34) In what time will 600*l.* amount to 856*l.* 10*s.* at $4\frac{1}{2}$ per cent. per annum?

(35) In what time will 498*l.* 6*s.* 8*d.* amount to 614*l.* 3*s.* 11*d.* at 3 per cent. per annum?

Case 9. When the principal, amount, and time are given, to find the rate per cent.

RULE.

1. Say, As the principal : is to the interest, for the whole time :: so is 100*l.* : to the interest for the same time.

2. Divide that interest by the given time, and the quotient will be the rate per cent required.

EXAMPLES.

(36) At what rate per cent. per annum will 600*l.* amount to 856*l.* 10*s.* in $9\frac{1}{2}$ years?

(37) At what rate per cent. per annum will 498*l.* 6*s.* 8*d.* amount to 614*l.* 3*s.* 11*d.* in $7\frac{3}{4}$ years?

QUESTIONS *for Exercise in the last Eight* CASES, *at leisure Hours.*

(38) Lent at Christmas 1804, the sum of 5000*l.* at $4\frac{1}{2}$ per cent. after which time I lent several sums at the same rate, and drew upon the borrower, as business required; viz. on Lady-day 1805, I drew for 185 guineas; on Midsummer-day following, I lent 500 moidores, and drew for 700*l.* and on Michaelmas day, in the same year, I lent 569*l.* 17*s.* I demand what cash the borrower owed me at that time.

(39) On the first of May 1805, I lent Ralph Newlands, per bill at one day's date, 500*l.* which I received back in the following partial payments; viz. on the 13th of May 50*l.* on the 4th of June 56*l.* on the 14th of July 44*l.* on the 23d ditto 50*l.* on the 18th of August 87*l.* on the 30th ditto 13*l.* on the 21st of September 30*l.* on the 18th of October 30*l.* on the 29th ditto 40*l.* on the 11th of November 50*l.* and on the 28th of December 50*l.* Now I demand to know what interest is due at 5 per cent. per annum?

(40) Lent to John Jameson, per bill dated 18th of Jan. 1805, payable one day after date, 878*l* 19*s.* 10*d.* which I received back in the following partial payments; viz. on the 27th of February 57*l.* 15*s.* 7*d.* on the 18th of March 37*l*, 14*s.* on the 29th of April 34*l.* 11*s.* on the 12th of May 136*l.* 15*s.* 7*d.* on the 19th of June 67*l.* 13*s.* 4*d.* on the 15th of July 15 guineas and 6*d.* on the 25th ditto 111*l.* 11*s.* 11*d.* on the 3d of Oc-

tober 78*l.* 7*s.* 4*d.* on the 19th of November 100*l.* on the 23d ditto 100*l.* and on the 30th of December received the balance of the principal. How much interest ought I to claim at 5 per cent.?

(41) Lent 109 guineas at 4 per cent. By the 18th of August, 1799, they were raised by the interest to so many moidores, abating half-a-crown; pray on what day did the bond bear date?

(42) If 100*l.* in 12 years be allowed to gain 39*l.* 19*s.* 8*d.* in what time will any other sum double itself at the same rate of interest?

(43) A bond was made on the 7th of August 1786, at 6 per cent. per annum, for 1114*l*, 10*s.* On the 11th of May, 1789, 140*l.* was paid off, and a fresh bond entered into for the remainder at $5\frac{1}{2}$ per cent. per annum; at the time the interest for this last was 21*l.* 16*s.* 8*d.* there was paid off 87*l.* 11*s.* 9*d.* The old bond being then taken up, a new one was given for the residue, which being paid off September 11, 1796, the bond-owner took no more than 1409*l.* 16*s.* 8*d.* in full payment: at what rate did he take interest per cent. per annum, upon the last renewal of the bond?

(44) It is proposed by an elderly person in trade, desirous of a little respite, to admit a sober industrious young man in the business; and, to encourage him, offers, that if his circumstances will allow him to advance 100*l.* his pay shall be 40*l.* a year; if he shall be able to put 200*l.* into the stock, he shall have 55*l.* a-year, and if 300*l.* he shall receive 70*l.* annually. In this proposal, what was allowed for his attendance simply; and what rate per cent. was allowed for his money?

(45) June 23d, 1745, bought 900*l.* of New South-Sea annuities, at $111\frac{3}{8}$ per cent. viz. the day before the closing the books, the brokerage of which is always 2*s.* 6*d.* per cent. on the capital, whether you buy or sell. The Midsummer dividend 2 per cent. became due and payable on the 10th of August following; by which time the rebellion growing considerable in the North, the said annuities were down at $92\frac{1}{2}$ per cent. In the general alarm, sold 400*l.* capital at that price; but continued the remainder, till a second, third, fourth, and fifth dividend, as before, came due; and on open-

ing the books on the 10th of August 1747, sold out at $102\frac{5}{8}$ per cent. Now reckoning I might have made 5 per cent. of my money, had I kept it out of the stocks, how stood this article in point of profit and loss?

XIX. COMPOUND INTEREST

IS that which arises both from principal and its interest, put together, as the interest becomes due, but not paid; the same interest is allowed upon that interest unpaid, as was upon the principal; so it becomes a part of the principal; and for which reason, it is called Interest upon Interest, or Compound Interest.

It is not lawful to let out money at Compound Interest: yet, in purchasing of annuities or pensions, and leases in reversion, it is usual to allow Compound Interest to the purchaser for his ready money, and therefore makes it necessary that it should be understood.

But as it may (as well as other cases of interest) be more conveniently performed by decimals, I shall only give the rule, and a few examples.

RULE.

1. Find the first year's interest as in case 1. Add that interest to the principal, which sum will become the second year's principal, and so on for any number of years.

2. Subtract the given principal from the last amount, and the remainder will be the interest required.

EXAMPLES.

(1) What is the compound interest of 600*l.* forborne 3 years, at 5 per cent. per annum?

(2) What is the amount of 150*l.* for 5 years, at 4 per cent. per annum, compound interest?

(3) What is the compound interest of 440*l.* 16*s.* for 4 years, 7 months, and 15 days, at 5 per cent. per annum?

Note.—When the interest is required for months and days besides years, you must find the interest for one year more than the number of years given, and, from that year's

interest, take parts for the months and days, which add to the last year's interest, and the sum will be the interest required.

XX. REBATE, or DISCOUNT,

IS the abating of so much on a debt, to be paid before it becomes due; which payment (rebate being deducted), if put out to interest for the same time, and rate per cent. per annum, would be equal to the sum first due.

RULE.

1. Find the interest of 100*l.* for the time given, and rate per cent. which interest add to 100*l.*

2. As that sum : is to the interest of 100*l.* or 100 :: so is the debt or sum proposed : to the rebate or present worth required. Or, subtract the rebate from the given sum, and the remainder will be the present worth, or money to be paid down.

EXAMPLES.

(1) What is the rebate of 420*l.* for 7 months, 6 days, at 5 per cent. per annum?

(2) What is the present worth of 100*l.* for 12 months, at 6 per cent.?

(3) What is the rebate and present worth of 600*l.* 10*s.* 6*d.* payable in 10 months, at 4 per cent. per annum?

(4) What is the discount of 890*l.* 16*s.* being due July 27, 1807, this being December 12, 1806, at 5 per cent. per annum?

(5) Sold goods to the value of 430*l* to be paid at two 4 months; that is, half at 4 months, and the other half at 8 months. What must be discounted for the present payment of the whole, discount being at 5 per cent. per annum?

(6) Suppose I have a legacy of 550*l.* left me on the 21st of May, 1806, but not to be paid till Chistmas-day following; what is the present worth, discount allowed at five per cent. per annum?

(7) What is the present worth of 3220*l.* payable as follows: 100*l.* at 3 months, 60*l.* at 5 months, and the

remainder at 6 months; discount at 6 per cent. per annum?

(8) Sold goods to the value of 400*l.* to be paid at three 3 months, as follows: viz. first $\frac{1}{3}$, second $\frac{1}{2}$, and the rest the third payment; what is the discount and present worth of the whole at $4\frac{1}{2}$ per cent. per annum?

(9) What ready money will discharge a debt of 360*l.* due at two 5 months; that is, $\frac{1}{5}$ at 5 months, and the rest at 10 months; discount at 3 per cent. per annum?

(10) What difference is there between the interest of 500*l.* at 5 per cent. per annum for 12 years, and the discount of the same sum, at the same rate, and for the same time?

(11) What ready money will discharge a debt of 13377*l.* 13*s.* 4*d.* due two years, three quarters, 25 days hence; discount at $4\frac{3}{8}$ per cent. per annum?

XXI. EQUATION OF PAYMENTS

IS when several debts are payable at different times, but it is mutually agreed, between debtor and creditor, that those several sums be all paid at once, and at such a time, as that neither party may be wronged thereby; this is called equating the time of payment. The common rule is as follows:

RULE.

Multiply the sum of each particular payment by its time, then add the products together, and divide the sum by the whole debt; the quotient, by this rule, is the equated time for the payment of the whole.

EXAMPLES.

(1) B. owes C. 600*l.* whereof 200*l.* is to be paid at 3 months, 150*l.* at 4 months, and the rest at 6 months; but they afterwards agreed the whole should be paid at once; required the time.

(2) A. bought of B. a quantity of goods which came to 460*l.* to be paid in the following manner, viz. 200*l.* at seven months, and the rest at five months; but after-

wards they agree to make one payment of the whole. I demand the equated time.

(3) C. owes D. a certain sum, which is to be discharged in the following manner, viz. $\frac{1}{2}$ at three months, $\frac{1}{3}$ in 4 months, and $\frac{1}{6}$ at 9 months; but they afterwards agree to have but one payment of the whole: the equated time is required.

(4) A debt is to be discharged thus, viz. $\frac{1}{4}$ present, $\frac{1}{4}$ at 4 months, $\frac{1}{4}$ at 5 months, and the rest at 6 months; what is the equated time for the whole?

(5) E. is indebted to F. 240*l.* which by agreement is to be paid at 5 months hence: but E. is willing to pay him 40*l.* down, provided he will give him a longer time for the payment of the remainder, which is agreed on: the time of payment is required.

XXII. SINGLE FELLOWSHIP,

OR,

Fellowship without Time,

IS when two or more persons join their stocks and trade together. To determine each person's particular share of the gain or loss, in proportion to his principal paid into the stock, observe the following

RULE.

As the sum of the several stocks : to the gain or loss :: so is each person's share in the stock : to his share of the gain or loss.

PROOF.

Add all the shares together, and that sum, if right, will be equal to the whole gain or loss.

EXAMPLES.

(1) Two merchants, A. and B. join in partnership: A. laid in 80*l.* B 60*l.* and they gain 28*l.* What is each man's share of the said gain?

(2) Three persons, C. D. and E. trade together, and make a

joint stock of 824*l.* and in three years' time they gained as much, and 70*l.* over; C.'s stock was 320*l.* D.'s 340*l.* I demand E.'s stock, and what each person gained by trading.

(3) Suppose four merchants, A. B. C. and D. join their stocks and trade together, of which A. puts in $\frac{1}{2}$, B. $\frac{1}{3}$, C. $\frac{1}{4}$, and D. $\frac{1}{5}$; but at the expiration of 12 months, they had the misfortune to lose 120*l.* What must each person suffer of the said loss?

(4) Three merchants. D. E. and F. in partnership together, with one common stock of 400*l.* gained as follows, viz. D. 30*l.* E. 48*l.* and F. 42*l.* What was each man's stock?

(5) Suppose the money and effects of a bankrupt amount to 2420*l.* 17*s.* 6*d.* and he is indebted as follows, viz. to A. 1000*l.* to B. 640*l.* to C. 900*l.* and to D. 842*l.* 16*s.* How must his effects be divided amongst them? or what share must each have?

QUESTIONS *for Exercise at leisure Hours.*

(6) A father ordered 500*l.* to be divided amongst his five sons, thus. Give A. said he, $\frac{1}{3}$, B. $\frac{1}{4}$, C. $\frac{1}{5}$, D. $\frac{1}{6}$, and E. $\frac{1}{7}$. Part this equitably amongst them, according to the father's intentions.

(7) Three persons purchase together a West-India sloop, towards which A. advanced $\frac{3}{8}$, B. $\frac{3}{7}$, and C. 140*l.* How much paid A. and B. and what part of the vessel had C.?

(8) A. and B. clear by an adventure at sea 50 guineas, with which they agree to buy a horse and chaise; whereof they were to have the use in proportion to the sums adventured, which was found to be A. 10; B. 7: they cleared 45 per cent. What money did each send abroad?

(9) A. and B. join their stocks, and vest them in brandy. A.'s stock was 19*l.* 19*s.* 8*d.* more than that of B. Now by selling out their commodity at 55*s.* per anker, A. cleared 74*l.* 11*s.* and B. just 50 guineas. The quantity of brandy dealt for is required, and the gain upon the anker.

(10) In an article of trade A. gains 14*s.* 6*d.* and his adventure was 35*s.* more than B.'s, whose share of the

profit is but 8*s.* 6*d.* What are the particulars of their stock?

(11) Three persons, A. B. and C. entered into joint trade; to which A. contributed 210*l.* B. 312*l.* They cleared 140*l.* whereof 37*l.* 10*s.* belongs of right to C. That person's stock and the several gains of the other two are required.

(12) A. and B. venturing equal sums of money, clear by joint trade 154*l.* By agreement A. was to have 8 per cent. because he spent time in execution of the project, and B. was only to have 5. What was allotted A. for his trouble?

(13) A. B. and C. are three horses belonging to different men, and are employed as a team to draw a load of wheat from Hertford for 30*s.* A. and B. are deemed to do $\frac{2}{7}$ of the work, A. and C. $\frac{3}{8}$, and B. and C. $\frac{3}{10}$ of it: they are to be paid proportionably. Do you know how to divide it as it should be?

(14) Bought 100 quarters of malt, meal, and oatmeal, together, for 142*l.* For every 5 bushels of malt I had 3 of meal; for every 8 of meal I had 7 of oatmeal. What did these cost me severally per bushel, the malt being half as dear again as the meal, and the meal being double the price of the oatmeal?

(15) In raising a joint stock of 400*l.* A. advanced $\frac{4}{13}$, B. $\frac{12}{11}$ of $\frac{3}{8}$, C. $\frac{5}{6}$ more the difference between A.'s adventure and B.'s; and D. the rest of the money. What did every one subscribe?

XXIII. DOUBLE FELLOWSHIP,

OR,

Fellowship with Time,

IS when each person's stock continues unequal time in company, so that a consideration must be made of the time, as well as of the stock.

RULE.

Multiply each person's stock by the time it has continued in trade, and proceed with the products, as with the particular stocks in Single Fellowship.

PROOF.

As in Single Fellowship.

EXAMPLES.

(1) Three merchants, A. B. and C. enter into partnership, thus: A. puts into the stock 240*l.* for 4 months, B. 120*l.* for 6 months, and C. 200*l.* for 8 months; with this joint stock they traffic, and gain 260*l.* It is required to find each person's share of the gain, proportionable to his stock and time of employing it.

(2) A ship's company take a prize, value 4000*l.* which they agree to divide amongst them according to their pay and time they have been on board; now the officers and midshipmen have been on board 4 months, and the sailors 3; the officers have 50*s.* a month; the midshipmen 40*s.* and the sailors 28*s.* There are also 4 officers, 8 midshipmen, and 120 sailors. I demand what is each person's share of the said prize?

(3) A. B. and C. rent a piece of land, for which they pay 40*l.* per annum; A. puts in 60 oxen for 4 months, B. 40 oxen for 5 months, and C. 30 oxen for the remainder of the year; what must each person pay of the said rent?

(4) Three merchants, A. B. and C. in partnership together for a year, put into one common stock as follows, viz. A. puts in 400*l.* and at 6 months' end withdraws 200*l.*; B. puts in 360*l.* and at 7 months' end 100*l.* more, but at the end of 9 months he takes out 120*l.*; C. puts in 190*l.* and at 8 months' end 110*l.* more, but at the end of 10 months he takes out 100*l.* They gain 460*l.* What is each man's share?

QUESTIONS *for Exercise at leisure Hours.*

(5) A. and B. in partnership equally divide the gain: A.'s money, which was 84*l.* 12*s.* 6*d.* lay for 19 months, and B.'s for no more than 7: the adventure of the latter is sought.

(6) A. for 9 months' adventure received 20*l.* B. for one of 7 months received 25 guineas, and C. for lying out

of his contributions 5 months had a title to 32*l.* The total of their adventures, multiplied into their respective times, was 640*l.* What then were the particulars?

(7) A. clears 13*l.* in 6 months, B. 18*l.* in 5 months, and C. 23*l.* in 9 months, with a stock of 72*l.* 10*s.* What then did the general stock amount to?

(8) X. Y. and Z. in company, make one common stock of 4262*l.* X.'s money was in 4 months, Y.'s 6 months, and Z.'s 9 months; they gained 420*l.* which was to be divided in the following manner, viz. $\frac{1}{2}$ of X.'s gain to be equal to $\frac{1}{3}$ of Y.'s, and $\frac{1}{3}$ of Y.'s gain to be equal to $\frac{1}{4}$ of Z.'s. Quere, what each person gained and put in?

(9) A. B. and C. in company; A. put in his share of the stock for 5 months, and laid claim to $\frac{1}{5}$ of the profits; B. put in his for 8 months; C. advanced 400*l.* for 7 months, and required on the balance $\frac{2}{3}$ of the gain: the stock of the other two adventurers is sought.

(10) A. and B. paid equally for a horse, Feb. 7, 1805; A. on the 10th took him a journey to the west, and returned on the 10th of June following: B. on the 2d of August took him into Scotland, and staid till Nov. 13, and thus concluded his service for this year. From Jan. 17th following, A. used him ten days; and, in six weeks after his return, employed him till April 30th. B. then rode him from May-day to Midsummer: A. had him from the 14th of July to 14 days after St. James's tide; B. on Sept. 30th, took him into Norfolk, and came back Oct. 19th: he then was sold for 7*l.* 10*s.* and they would have the money parted equally between them, viz. in proportion to the use each made of their steed.

XXIV. BARTER

IS the changing of one commodity for another, and informs us how to proportion the value of any goods, so that neither party may sustain loss. And if the commodities exchanged are not of equal value, the defect is supplied with money.

RULE.

1. Find the value of that commodity, whose quantity is given: then find what quantity of the other, at the given rate, you can have for the aforesaid value; which quantity will be the answer.

2. When one has goods at a certain price ready money, but in barter advances it to something more, say, As the ready money price of the one : is to its bartering price :: so is the ready money price of the other to its bartering price: then the quantity of the latter commodity may be found, either from the ready money or bartering price.

EXAMPLES.

(1) How much sugar at 1*l.* 10*s.* per cwt. must be given in barter, for 4 cwt. of tea, at 12*s.* per pound?

(2) How many yards of cloth, at 18*s.* per yard, must I give for 45 yards of shalloon, at 16*d.* per yard?

(3) A. and B. barter: A. has 30 cwt. of prunes at 6*d.* per lb. ready money, but in barter will have 7½*d.* per lb. B. has hops worth 36*s.* per cwt. ready money. What ought B. to rate his hops in barter, and what quantity must be given for the 30 cwt. of prunes?

(4) A. has tea at 8*s.* 6*d.* per lb. ready money, but in barter will have 10*s.* per lb. B. has tobacco worth 18*d.* per lb. ready money. How must B. rate his tobacco per lb. that his profit may be equivalent with A.'s?

QUESTIONS *for Exercise at leisure Hours.*

(5) A. has currants worth 4*d.* per lb. but in barter charges 6*d.* and also requires one half of that in ready money; B. has candles worth 6*s.* 8*d.* the dozen, and he, in barter, charges but 7*s.* Should these persons deal together for the value of 20*l.*, how much will A. have gained of B.?

(6) A. lets B. have a hogshead of sugar, weight 18 cwt. worth 31*s.* for 42*s.* per cwt. one-third of which he is to pay in cash; B. has paper worth 14*s.* per ream, which it is agreed shall bear no more than 15*s.* 6*d.* At that rate, and barter for the rest, how stood the account?

(7) A. has kerseys at 4*l.* 5*s.* a piece ready money; in barter they are charged by him at 5*l.* 6*s.* each, and $\frac{1}{2}$ of that required down; B. has flax at 3d per lb. How ought he to rate it not to be hurt by the extortion of A.?

(8) A. has 50 broad cloths, at 11*l.* 10*s.* a piece, but in change required 13*l.* taking wool at 2*s.* 6*d.* per stone, of B. in return, that was really worth but 4*s.* 2*d.* a tod. The question is, how many sacks of wool will pay for the cloth, and which of the dealers has the advantage in the bargain.

(9) A. with an intention to clear 30 guineas on a bargain with B. rates hops at 16*d.* per lb. that stood him in 10*d.* B. apprised of that, set down malt which cost 20*s.* a quarter at an adequate price. How much malt did they contract for?

(10) A. in order to put off to B. 720 ells of damaged Holland, worth 5*s.* an ell, at 6*s.* 8*d.* proposes, in case he has half the value in money, to allow B. a discount of 10 per cent. The rest A. is to take out in saffron, which B. apprised of the whole management, rates in justice at 30*s.* per pound. What was it really worth in ready money, and what quantity of saffron was he to deliver on the change?

(11) A. has 100 reams of paper, at 8*s.* ready money, which in barter he sets down at 10*s.* B. sensible of this, has pamphlets at 6*d.* a piece ready money, which he adequately charges, and insists, besides, on $\frac{1}{4}$ of the price of those he parts with in specie: what number of the books is he to deliver in lieu of A.'s paper? What cash will make good the difference? and how much is B. the gainer by this affair?

(12) A. and B. barter; A. has 140 lb. 11 oz. of plate, at 6*s.* 4*d.* the ounce, which in barter he rates at 7*s.* 2*d.* an ounce, and allows a discount on his part to have $\frac{1}{7}$ of that in ready specie; B. has tea worth 9*s.* 6*d.* the lb. which he rates at 11*s.* 2*d.* When they come to strike the balance, A. received but 7 cwt. 2 qrs. 18 lb. of tea. What discount did A. allow B., which of them had the advantage, and how much, in an article of trade thus circumstanced?

(13) A. and B. barter; A. has 14 cwt. 2 qrs. 25 lb. of Farnham hops, at 2*l.* 19*s.* per cwt. but in barter insists on

3 guineas; B. has wine worth 6*s.* per gallon, which he raises in proportion to A.'s demand: on the balance, A. received but a hogshead and a half of wine. What had he in ready money?

XXV. LOSS AND GAIN

IS a rule by which we discover the gain or loss by any parcel of goods, and so instructs us how to raise or fall the price of any commodity in such proportions, that neither our gain may be so exorbitant as to injure our customers, nor our loss so great as to impoverish ourselves; which is generally at so much per cent.

In this rule there are a great variety of examples, all of which may be easily solved, with a little consideration, by the following proportion:

Case 1. When the quantity lost and gained of the whole is given, to find the value of any part thereof.

RULE.

Say, As the whole quantity of goods : is to the sum of the whole cost and proposed gain :: so is any part of said goods : to the price they must be sold for.

Case 2. When the proposed gain or loss is at so much per cent., make 100*l.* with the gain or loss added to it, your second term.

EXAMPLES.

(1) Bought 240 yards of cloth, at 14*s.* 6*d.* per yard, and sold it again at 18s. per yard. What did I gain by the whole?

(2) Suppose I give 46*l.* for 9 cwt. 2 qrs. 18 lb. of sugar, at what rate must I sell it per lb. to gain 12 guineas by the whole?

(3) If I buy tea at 8*s.* 6*d.* per lb. and sell it again for 10*s.* 6*d.* what is the gain per cent.?

(4) If by selling cloth at 5*s.* per ell, I gain 8*l.* per cent, what shall I gain per cent. if I sell the ell at 6*s.* 3*d.*

(5) At 5*s.* per dozen, I gain 7*l.* 10*s.* per cent. how much shall I gain per cent. if I sell the dozen at 5*s.* 9*d.*

(6) A Manchester tradesman going to a fair, sold fustians for 11*s.* 6*d.* the end, wherein was gained 15*l.* per. cent. but, seeing no other tradesman had so good, raised them at the latter end of the fair to 12*s.* the end. I demand what he gained per cent, by this last sale.

(7) Suppose I sell 500 deals at 15*d.* per piece, and 9*l.* per cent. loss, what do I lose by the whole quantity?

(8) Suppose I sell 1 cwt. of hops, for 6*l.* 15*s.* and gain 25*l.* per cent. what would have been the gain per cent. if I had sold them for 8*l.* per cent.?

(9) If by selling hops at 3*l.* 10*s.* per cwt. the planter clears 30 per cent. what was his gain per cent. when the same goods sold for 4*l.* and a crown?

(10) Sold a repeating watch for 50 guineas, and by so doing lost 17 per cent. whereas I ought in dealing to have cleared 20 per cent. Then how much was it sold under the just value?

QUESTIONS *for Exercise at leisure Hours.*

(11) If by sending pewter to Turkey, and parting with it at $35\frac{2}{3}$*d.* per lb. a merchant clears cent. per cent. what does he clear in Holland, where he disposes of the cwt. for 8*l.*?

(12) Bought hose in London, at 4*s.* 3*d.* per pair, and sold them afterwards in Dublin at 6*s.* Now, taking the charges at an average to be 2*d.* per pair, and considering that I must lose 12 per cent. by remitting my money home again, what do I gain per cent. by this article of trade?

(13) If my factor at Leghorn returns me 800 barrels of anchovies, each weighing 14 lb. net, worth $12\frac{1}{2}$*d.* per lb. in lieu of 7490 lb. of Virginia tobacco, and if I find that I have gained after the rate of 17*l.* per cent. by the said consignment; pray how was my tobacco invoiced per lb. to the factor? or what was the prime cost?

(14) Bought comfits to the value of 41*l.* 3*s.* 4*d.* for 3*s.* 1*d.* per lb. It happened that so many of them were damaged in carriage, that by selling what remained good at 4*s.* 6*d.* per lb. my returns were no more than 34*l.* 2*s.* 6*d.* Pray how much of these goods were spoiled, and what did this part stand me in?

(15) A stationer sold quills at 11*s.* per thousand, by which he cleared $\frac{3}{8}$ of the money; but they growing scarce, he raised them to 13*s.* 6*d.* per thousand. What might he clear per cent. by the latter price?

(16) A. had 15 pipes of Malaga wine, which he parted with to B. at $4\frac{1}{3}$ per cent. profit, who sold them to C. for 38*l.* 11*s.* 6*d.* advantage; C. made them over to D. for 500*l.* 16*s.* 8*d.* and cleared thereby $6\frac{1}{2}$ per cent. What did this wine cost A. per gallon?

(17) Laid out in a lot of muslin 480*l.* 12*s.*, upon examination of which two parts in seven proved damaged; so that I could make but 5*s.* 6*d.* per yard of the same; and by so doing, find I lost 48*l.* 18*s.* by it. At what rate per ell am I to part with the undamaged muslin, to make up my loss?

XXVI. ALLIGATION MEDIAL

IS when the price and quantity of several commodities are given to be mixed, to find the mean price of that mixture.

RULE.

As the whole composition : is to its total value :: so is any part thereof : to its mean price.

PROOF.

Find the value of the whole mixture at the mean rate; and if it agree with the total value of the several quantities at their respective prices, the work is right.

EXAMPLES.

(1) A wine merchant mingles 14 gallons of mountain wine at 8*s.* per gallon, with 12 gallons at 6*s.* per gallon, 10 gallons of sherry at 7*s.* per gallon, 20 gallons of white wine at 4*s.* per gallon, and 8 gallons

of canary at 9*s.* per gallon. How may he sell this mixture per gallon?

(2) With 13 gallons of canary, at 6*s.* 8*d.* per gallon, I mingle 20 gallons of white wine, at 5*s.* per gallon; and to these add 10 gallons of cider, at 3*s.* per gallon. At what rate must I sell a quart of this mixture, so as to clear 10 per cent.?

XXVII. ALLIGATION ALTERNATE

IS when the rates of several commodities are given, to find such quantities of them as, being mixed together, shall bear a price propounded.

RULE.

1. The rates (if not already) must all be reduced to one denomination.

2. Set down the rates, or prices, in a column under one another, and the mixed or mean rate on the left hand of these.

3. Connect or link together the several rates, so that every one less than the mean be linked with some one greater, or with as many as you please that are greater, and every great with one less, or with as many less as you please.

4. Take the difference between each price and the mean rate, and set them alternately; and if only one difference stand against any rate, it will be the quantity belonging to that said rate; but if there are several, then their sum will be the quantity; which quantities are the answer for that rate against which they stand.

EXAMPLES.

(3) To mix gold of 18 carats fine with that of 23 carats fine, of 19, and of 16 carats fine, so that the composition may be 20 carats fine; what quantity of each must be taken?

(4) A grocer would mix a quantity of sugar at 10*d.* per lb. with other sugar at $7\frac{1}{2}$*d.* 5*d.* and $4\frac{1}{2}$*d.* per lb. intending to make up a commodity worth 6*d.* per lb. In what proportion is he to take of those sugars?

XXVIII. ALLIGATION PARTIAL

IS when the price of each simple is given, also the quantity of one of them, and the mean rate, to find the several quantities of the rest in proportion to that given.

RULE.

1. Take the difference between each price and the mean rate as in the last rule.

2. As the difference of that simple whose quantity is given : is to the known quantity :: so is any other difference : to the quantity of its opposite name.

EXAMPLES.

(5) How much tea at 6*s.* 6*d.*, 7*s.* 6*d.* and 9*s.* per lb. must be taken to be mixed with 36 lb. at 12*s.* per lb. that the mixture may be worth 8*s.* per lb.

(6) A tobacconist has by him 120 lb. of fine Oronoko tobacco, worth 2*s.* 6*d.* a pound; to this he would mix York river ditto at 20*d.* and other inferior tobaccos at 18*d.* and 15*d.* a pound, as will make up a mixture answerable to 2*s.* a pound. What will this parcel weigh?

XXIX. ALLIGATION TOTAL

IS when the price of each simple is given, as also the mean rate, and what quantity of the compound, to find how much of each sort will make that quantity.

RULE.

1. Take the difference between each price and the mean rate, as before.

2. Say, as the sum of these differences : is to the whole quantity of the mixture :: so is each particular difference : to its particular quantity.

EXAMPLES.

(7) How much gold of 16, of 18, and 23 carats fine, must be mixed together, to form a composition of 60 oz. of 20 carats fine?

(8) A grocer has by him 4 sorts of green tea, viz. of 5*s.* 6*s.*, 8*s.* and 9*s.* per lb.; out of these he is inclined to mix up a canister, containing net, a hundred and a half, so as to make the commodity worth 7*s.* per pound. In what proportion must those teas be taken?

XXX. EXCHANGE

IS the receiving in one country for the value paid in another.

The par of exchange is always fixed and certain, it being at the intrinsic value of any foreign money compared with sterling: but the course of exchange between any two countries rises and falls upon various occasions.

But as it would be both needless and endless to write of every kind of exchange, so I shall only give a few examples of the exchange of England with some of the chief countries of Europe.

Exchange is either performed by Sec. XII. or XV. and sometimes most expeditiously by the latter.

1st. With FRANCE.

They keep their accounts at Paris, Lyons, and Rouen, in livres, sols, and deniers, and exchange by the crown of three livres Tournois, or 60 sols French, and give pence sterling, more or less, for this exchange crown, which is equal to 4*s.* 6*d.* at par.

12 Deniers	make one	Sol.
20 Sols		Livre.
3 Livres		Crown.

Case 1. To change French money into sterling.

RULE.

As 1 crown : is to the given rate :: so is the given French sum : to the sterling required; or by the rules given in Practice.

Case 2. To change sterling money into French.

RULE.

As the rate of Exchange : is to 1 crown :: so is the sterling sum : to the French required.

Note.—The same rule must be observed with most of the following countries.

EXAMPLES.

(1) What sterling money must be paid in London, to receive in Paris 500 crowns; exchange at 56*d.* per crown?

(2) How many crowns must be paid at Paris, to receive in London 116*l.* 13*s.* 4*d.* the exchange at 56*d.* per crown?

(3) Change 640 crowns, 12 sols, 8 deniers, at $54\frac{1}{2}$*d.* per crown, into sterling.

(4) Change 145*l.* 7*s.* $7\frac{1}{2}$*d.* sterling, into French crowns, exchange at $54\frac{1}{2}$*d.* per crown.

2d. With SPAIN.

They keep their accounts at Madrid, Cadiz, and Seville, in dollars, rials, and maravedies, and exchange by the piece of eight, which is equal to 4*s.* 6*d.* at par.

4 Maravedies vellon, or	make one	Quarta.
$2\frac{1}{2}$ Maravedies plate		
$8\frac{1}{2}$ Quartas, or		Rial vellon.
34 Maravedies vellon		
16 Quartas, or		Rial of plate.
34 Maravedies plate		
8 Rials of plate		Piece of eight or dollar.

N. B. A rial vellon is $\frac{17}{32}$ of a rial of plate, and $\frac{17}{256}$ of a piastre.

EXAMPLES.

(5) Change 856*l.* 6*s.* 8*d.* into Spanish money, exchange at 56*d.* per piece of eight.

(6) If I pay in Seville 1426 pieces of eight, 4 rials, 26 maravedies, what may I draw my bill for on London, exchange at $54\frac{1}{4}$*d.* per piece of eight?

3d. With Italy.

In Italy they keep their accounts, at Genoa and Leghorn, in livres, sols, and deniers, and exchange by the piece of eight or dollar, which is equal to 4*s.* 6*d.* at par.

12 Deniers	make one	Sol.	
20 Sols		Livre.	
5 Livres		Piece of eight at	Genoa.
6 Livres			Leghorn.

At Florence the exchange is by ducatoons, and at Venice by ducats, divided as follows, viz.

6 Soldi	make one	Gross.
24 Grosses		Ducat.

EXAMPLES.

(7) Genoa is indebted to London 640 dollars. For how much sterling may London draw on Genoa, the exchange at 52*d.* per dollar?

(8) If a merchant remit 138*l.* 13*s.* 4*d.* sterling to Leghorn, how many dollars will he receive there, the exchange at 52*d.* per dollar?

Note.—In St. George's bank, at Genoa, accounts are kept in piastres or pezzoes, which are divided into soldi and denari, as the pound sterling.

But some merchants keep their accounts in lires, or liras, soldi, and denari, divided as before. This money is only one fifth in value of the bank money.

(9) Change 8644 pez. into sterling money, exchange at 47½*d.* per pezzo.

(10) London is indebted to Genoa 1710*l.* 16*s.* 4*d.* For how many pezzoes may Genoa draw on London, the exchange at 47½*d.* per pez.?

4th. With PORTUGAL.

They keep their accounts in Lisbon, Oporto, &c. in reas, and exchange on the milrea; and London gives from 5*s.* to 5*s.* 6*d.* for the same.

400 Reas	make one	Crusadoe.
1000 Reas		Milrea.

EXAMPLES.

(11) A merchant at Lisbon being desirous to remit to his correspondent at London 4760 milreas, exchange at 64*d.* per milrea, how much sterling must be paid in London?

(12) How many milreas will 1566*l.* 6*s.* 8*d.* amount to, exchange at 64*d.* per milrea?

5th. With HOLLAND, FLANDERS, and GERMANY.

In these places, their accounts are kept sometimes in pounds, shillings, and pence, as in England; and sometimes in guilders, stivers, and pennings. The money of Holland and Flanders is distinguished by the name of Flemish, and the exchange is made with London, from 30 to 38 shillings Flemish, per pound sterling.

8 Pennings	make one	Groat.
2 Groats, or 16 Pennings		Stiver, or two-pence.
6 Stivers		Schilling.
20 Stivers		Florin, or guilder.
2½ Florins		Rix dollar.
6 Florins		Pound Flemish.
5 Guilders		Ducat.

To change Flemish money into sterling; and, on the contrary, sterling into Flemish, is the same with that of France; only what was French there will be Flemish here.

To reduce Flemish pounds, shillings, and pence, into guilders.

RULE.

Reduce them into pence Flemish, then divide by 40 (because 40*d.* are equal to one guilder), and the quotient will be guilders; and the remainder (if any) divide by 2 (because 2*d.* is equal to one stiver), and the quotient will be stivers.

EXAMPLES.

(13) A merchant in Rotterdam remits 564*l.* 10*s.* 6*d.* Flemish, to be paid in London. How much sterling money must he draw for, exchange at 34*s.* 4*d.* per £. sterling?

(14) Suppose a merchant delivers in London 328*l.* 16*s.* 11¾*d.* to receive the value at Amsterdam in Flemish money; how many pounds must he receive there, the exchange at 34*s.* 4*d.* Flemish per £. sterling.

(15) What may I draw my bill for on London, if I pay in Antwerp 4200 guilders, 12 stivers, 8 pennings, exchange at 33*s.* 3*d.* Flemish per £. sterling?

(16) If I pay in London 421*l.* 2*s.* 3*d.* how many guilders may I draw my bill for at Antwerp, exchange at 33*s.* 3*d.* Flemish per £. sterling?

(17) Exchange 242*l.* 13*s.* 6*d.* Flemish into guilders, stivers, &c.

6th. To change CURRENT MONEY into BANCO.

RULE.

As 100 with the agio added to it : is to 100 banco :: so is any given sum current : to the banco required.

EXAMPLE.

(18) Change 495 guilders, 18 stivers current, into banco florins, agio 5 per cent.

2. To change banco into current money.

RULE.

As 100 guineas banco : is to 100 with the agio added to it :: so is the banco given : to the current required.

EXAMPLE.

(19) Change 470 guilders, 8 stivers banco, into current, agio at 5⅓ per cent.

Note.—The bank money is worth more than the current: their difference is called agio, and is from three to six per cent. in favour of the bank.

7th. With VENICE.

Money of exchange here is always understood to be that of ducats in bank, which is imaginary, 100 of which make 120 ducats current money; so that the difference betwixt bank and current money is an agio of 20 per cent. though the brokers have invented another agio to be added, which is more or less according to bargain.

The course of exchange of a ducat of the bank of Venice is from 45*d.* to 50*d.* sterling.

EXAMPLE.

(20) Venice draws on London for 420 ducats, 10 sols, 8 den. banco, exchange at $47\frac{6}{8}$ per ducat. How much sterling will pay the draft?

8th. With POLAND and PRUSSIA.

Dantzic and Konigsberg exchange with London by way of Amsterdam and Hamburgh; 270 Polish grosh being = 1*l.* gross banco in Holland; 110 Polish grosh being = 1 rix-dollar, banco of Hamburg.

18 Phenningen	make one	Grosh.
3 Grosh		Ditkin.
2 Ditkins		Sixter.
3 Sixters		Tymph.
$7\frac{1}{2}$ Grosh		Arch de halber.
4 Arch de halbers		Florin or guilder.
3 Florins or guilders		Current } Dollar.
4 Guilders		Specie } Dollar.

EXAMPLES.

(21) Change 4684 florins into sterling money, 270 groshi Poli, per pound Flemish, and 34*s.* 4*d.* Flemish per £. sterling.

(22) Change 390*l.* sterling into florins, the exchange being 33*s.* 4*d.* Flemish per £. sterling, and 270 groshi Poli per £. Flemish.

9th. With RUSSIA.

3 Copecs	make one.	Altine.
10 Copecs		Grievener.
25 Copecs		Polpoliton.
2 Polpolitons		Poltin.
2 Poltins		Ruble.
2 Rubles		Ducat.

The Russian rubles are converted into florins, current money of Amsterdam, and the current into bank money, according to the agio of three or five per cent. and bank money into sterling, according to the course of exchange between England and Amsterdam.

EXAMPLE.

(23) In 6420 rubles, 42 copecs, exchange 122 copecs per rix-dollar current, agio 3 per cent. and 34*s.* 6*d.* Flemish, per £. sterling, how much sterling money?

10th. With IRELAND.

In Ireland they keep their accounts in £. *s.* and *d.* Irish, divided as in England; but having no coins of their own, they are supplied by the different countries with which they traffic.

The par of exchange between England and Ireland is 100*l.* sterling for 108*l.* 6*s.* 8*d.* Irish, or 1*s.* English = 13*d.*

The course of exchange is from 5 to 12 per cent. according to the balance of trade.

EXAMPLES.

(24) Dublin draws upon London for 740*l.* 14*s.* 6*d.* Irish, exchange at 12 per cent. How much sterling must London pay Dublin to discharge this bill?

(25) London remits to Ireland 651*l.* 14*s.* 11¼*d.* sterling, How much Irish must London be credited, exchange at 12 per cent.?

11th. With AMERICA and the WEST INDIES.

In exchange with our colonies in America and the West Indies, accounts are kept, and the money divided as in England: their money is called currency.

The scarcity of cash obliges them to substitute a paper-currency for carrying on their trade; which, being subject to casualties, suffers a very great discount for sterling, in the purchase of bills of exchange.

EXAMPLES.

(26) Philadelphia is indebted to London 1474*l.* 16*s.* currency. What sterling may London reckon to be remitted when the exchange is 64 per cent.?

(27) London receives a bill of exchange from Philadelphia, for 943*l.* 17*s.* 5¼*d.* sterling. For how much currency was London indebted, exchange being at 64 per cent.?

(28) London consigns to Jamaica goods, per invoice, amounting to 640*l.* 16*s.* 9*d.* which are sold for 987*l.* 12*s.* currency. What sterling ought the factor to remit, deducting 5 per cent. for commission and charges, and what does London gain per cent. upon the adventure, suppose the exchange at 30 per cent.?

(29) Jamaica is indebted to London 1470*l.* 12*s.* 8*d.* sterling. With how much currency will London be credited at Jamaica, when the exchange is 136½ per cent.?

A few EXAMPLES *for Exercise.*

(30) Amsterdam changes on London at 34*s.* 4*d.* per £. sterling, and on Lisbon at 52*d.* Flemish, for 400 rees. How then ought the exchange to go between London and Lisbon?

(31) A. at Paris draws on B. of London 1200 crowns, at 55*d.* sterling per crown; for the value whereof, B. draws again on A. 56*d.* sterling per crown, besides commission ½ per cent. Did A. get or lose by this transaction, and what?

(32) V. of Amsterdam draws on X. of Hamburg, at 67*d.* Flem. per dollar, of 32 sols Lubeck; and on Y. of Nuremberg, at 70*d.* Flemish per florin, of 65 cruitzers current. If V. has orders to draw on X. in order to remit to Y. at the said prices, how would run the exchange between Hamburg and Nuremberg?

(33) M. of Amsterdam orders N. of London to remit O. of Paris, at 54*d.* sterling per crown, and to draw on P. of Antwerp for the value, at 33½*s.* Flem. per £. sterling; but as soon as N. received the commission, the exchange was on Paris at 54½*d.* per crown. At what rate of exchange ought N. to draw on P. to execute his orders, and be no loser?

(34) London changes with Amsterdam on par at 33*s.* 4*d.* Flem. per £. Amsterdam changes on Middleburg, at 2 per cent. How stands the exchange between London and Middleburg?

(35) Q. of Rotterdam remits to R. of Paris 2000 crowns, at 91*d.* Flem. per crown, and double usance, or two months, and pays $\frac{3}{20}$ per cent. brokerage, with orders to remit him again the value at 93*d.* per crown, allowing at the same time ½ per cent. for provision. What is gained per cent. per annum, by a remittance thus managed?

(36) A. of Amsterdam owes B. of Paris 2000 florins of current specie, which he is to remit him by order, the exchange at 90½*d.* Flemish per crown, of 60 sols Tournois, the agio of the bank being four per cent. better than specie; but when it was to be negotiated, the exchange was down at 89½*d.* per crown, and the agio raised to five per cent. What did B. get by this turn of affairs?

XXXI. Comparison of Weights and Measures

IS when the weights or measures of different countries are compared together, and is a rule of great importance to the merchant, and very necessary to be acquainted with.

Case 1. When it is required to find how many of the first sort of weight or measure mentioned in the question, are equal to a given quantity of the last.

RULE.

1. Place the numbers alternately, beginning at the left hand, and let the last number stand on the left hand.

2. Multiply the first rank continually together for a dividend, and the second for a divisor.

EXAMPLES.

(1) If 100 lb. of London be equal to 113 lb. of Marseilles, and 100 lb. at Marseilles be equal to 81 lb. of Amsterdam; how many pounds at London are equal to 60 lb. of Amsterdam?

(2) If 104 lb. of English be equal to $84\frac{1}{2}$ lb. of Geneva, and 100 lb. of Geneva be equal to 108 lb. at Rouen; how many pounds English are equal to 64 lb. of Rouen?

(3) Suppose 100 yards English to be equal to 78 ells French, and 78 ells French equal to $133\frac{1}{3}$ ells at Amsterdam; how many yards English are equal to 100 ells at Amsterdam?

(4) If 100 canes of Genoa be equal to $191\frac{1}{3}$ ells of England, and 78 ells of England be equal to $131\frac{5}{7}$ of Brussels; how many canes of Genoa are equal to 100 ells of Brussels?

Case 2. When it is required to find how many of the last sorts, of weight or measure mentioned, are equal to a given number of the first.

RULE.

1. Place the numbers alternately, beginning at the left hand as before, and set the last number on the right hand.

2. Multiply the first row for a divisor, and the other for a dividend.

EXAMPLES.

(5) Suppose 100 lb. of Portugal be equal to 92 lb. of Antwerp, and 100 lb. of Antwerp be equal to 110 lb. of Lyons; how many pounds at Lyons are equal to 60 lb. of Portugal?

(6) If 74 yards of English be equal to 100 brasses of Florence, and 100 brasses of Florence be equal to 50 canes of Marseilles; how many canes of Marseilles are equal to 100 yards English?

XXXII. POSITION,

OR,

THE RULE OF FALSE,

IS so called, because we suppose some uncertain or false numbers; in order that, by reasoning from them, according to the nature thereof, we may, by those false supposed numbers, find the true number sought.

This Rule is divided into two parts, commonly called the Single Rule and Double Rule.

SINGLE POSITION.

By Single Position are answered all such questions as require only one supposition to discover the true result.

RULE.

Make choice of your position: work with that supposition, according to the nature of the question, as if it were the true number; and if you find, after ordering your position, the result either too much or too little, you may then find the true answer, by this proportion, viz.

As the result of your position : is to the position :: so is the given number : to the number sought.

PROOF.

Add the several parts of the sum together, and if the sum agree with the given number, it is right.

EXAMPLES.

(1) Three persons, A. B. and C. conversing respecting their ages; B. said to A. I am as old and half again as old as you: then said C. to B. I am twice as old as you: now said A. to them both, I am sure, if our ages be added together, the sum will be 132. I demand each man's age.

(2) A man overtaking a maid driving a flock of geese, said to her, Where are you going with these 40 geese? No, Sir, said she, I have not forty; but if I had as many more, half as many more, and 10 geese besides, I should have 40. How many geese had she?

(3) A. B. C. and D. were in company together: A. told C. that he was older than him by 4 years; B. told them that he was as old as both of them together, and 9 years older; D. hearing them, said, I am just 45 years old, and that is equal to the sum of your ages added together. How old was each of them severally?

(4) Three persons, viz. Andrew, Benjamin, and Christopher, are to go a journey of 469 miles; of this journey Andrew is to go a certain number of miles unknown; Benjamin is to go three times as many miles as Andrew, and one league more: and Christopher is to go twice as many miles as Benjamin, and 16 miles more. How many miles must each of these persons travel severally?

(5) Admit three merchants, A. B. and C. to build a ship, which costs them 2000*l.* of which A. paid a certain part unknown; B. paid $3\frac{1}{2}$ as much, wanting 45*l.* 15*s.*; and C. paid as much as both A. and B. together, and 26*l.* 10*s.* more. How much did each person pay?

(6) A cistern, with three unequal cocks, contains 60 pipes of water; the greater cock will empty the cistern in one hour, the second in two, and the third in three. In what time will they empty the cistern, supposing they all be set open at once?

(7) A general being asked the number of men his army consisted of, answered that $\frac{3}{4}$ of $\frac{1}{2}$ amounted to 900. What number of men had he?

(8) A schoolmaster being asked how many scholars he had, answered, If I had as many, $\frac{1}{2}$ as many, $\frac{1}{3}$ as many, and $\frac{1}{4}$ as many, I should have 333. How many had he?

XXXIII. DOUBLE POSITION

IS when two suppositions are used; and if we miss in both, as it generally happens, observe the nature of the errors, whether they be greater or less than the given number; and accordingly they must be made use of thus:

RULE.

1. Place the error against its respective position, and multiply them crosswise.

2. If the errors be alike, that is, both greater, or both less than the given number, take their difference for a divisor, and the difference of their products for a dividend.

But if unlike, that is, one too much, and the other too little, then take their sum for a divisor, and the sum of their products for a dividend; the quotient will be the answer.

EXAMPLES.

(1) A gentleman has two horses of good value, and a saddle worth 50*l.* which, if set on the back of the first horse, will make his value double that of the second; but if set on the back of the second horse makes his value triple that of the first horse. I demand the value of each horse.

(2) Double my money for me, said A. to B. and I will give you 6*d.* out of the stock: with the remainder he applied in the like manner to C. with equal success, and gave him also 6*d.* He repeated this proposal to D. and then 6*d.* was all he had to give. Pray, what sum had he to begin with?

(3) Three gentlemen, A. B. and C. playing at hazard together, the money staked was 112 guineas, but disagreeing, each seized as many as he could; A. got a certain quantity, B. as many as A. and 16 more; but C. got only a sixth part of their sum. How many had each?

(4) A boy stealing apples was taken by Mad Tom, and to appease him gave half he had, and Tom gave him back 10; in his return home he was met by Raving Ned, who took from him one half of what he had left, and returned him back 4; after that, unluckily, Positive Jack met him, when he gave him one half of what he had left, and he returned him back 1; at last getting safe away, he found he had 18 left. How many had he at first?

(5) A son asked his father how old he was; his father replied, Your age is now $\frac{1}{5}$ of mine; but 4 years ago, your age was only $\frac{1}{7}$ of what mine is now. What were their ages?

(6) The head of a certain fish is nine inches long, the tail as long as the head and half the body, and the body is as long as both the head and the tail. I demand the whole length of the said fish.

(7) To find a number, which, if added to itself, and the sum multiplied by the same, and the same number still subtrated from the product; and, lastly, the remainder divided by the same; that it may produce 13.

QUESTIONS *for Exercise at leisure Hours.*

(8) When first the marriage-knot was ty'd
Betwixt my wife and me,
My age did hers as far exceed
As three times three does three;
But when ten years, and half ten years,
We man and wife had been,
Her age came up as near to mine
As eight is to sixteen.

Ladies, or gentlemen, now tell, I pray,
What were our ages on the wedding-day?

(9) A gentleman finding several beggars at his door, gave to each four-pence, and had sixteen-pence left; but if he had given to each sixpence, he would have wanted twelve-pence. How many beggars were there?

(10) To find a number which, being multiplied by 3, subtract 5 from the product; and the remainder divided by 2, if the number sought be added to the quotient, that the sum may be 40.

(11) Two companions having a parcel of guineas, A. said to B. "If you will give me one of your guineas, I shall have as many as you will have left."—"Nay," replied B. "if you will give me one of your guineas, I shall have twice as many as you will have left." How many guineas had each of them?

(12) A son asked his father how old he was? His father answered him thus: If you take away 5 from my years, and divide the remainder by 8, the quotient will be $\frac{1}{3}$ of your age; but if you add 2 to your age, and multiply the whole by 6, and then subtract 7 from the product, you will have the number of the years of my age. What was the age of the father and son?

(13) Two men have a mind to purchase a house rated at 1200*l.* A. said to B. If you give me $\frac{1}{3}$ of your money, I can purchase the house alone; but B. replied to A. If you will give me $\frac{1}{4}$ of yours, I shall be able to purchase the house. How much money had each of them?

(14) Suppose the number 50 were to be divided into two parts, so that the greater part divided by 7, and the lesser multiplied by 3, the sum of this product, and the former quotient, might make the same number proposed, which was 50.

(15) A certain man hired a labourer on this condition, that for every day he worked he should receive 12 pence, but for every day he was idle he should be fined 8 pence. When 390 days were passed, neither of them were indebted to one another. How many days did he work, and how many days was he idle?

(16) A person being asked how old he was, answered, if I quadruple $\frac{2}{3}$ of my years, and add $\frac{1}{2}$ of them $+$ 50 to the product, the sum will be so much above 100 as the number of my years is now below 100.

(17) A certain person bought two horses, with the trappings, which cost 100*l.*; which trappings, if laid on the first horse A. both the horses will be of equal value; but if the trappings be laid on the other horse, he will be double the value of the first. How much did the horses and trappings cost?

(18) A young gentleman, at the age of 21 years, was told by his guardian, that his fortune consisted, in cash, of 7400*l.* and that his father died when he was but 10 years old: And for the money your father left, said the guardian, I have allowed you 5 per cent. per ann. for simple interest, only I have deducted 100*l.* per ann. for your education, &c. What was the son's fortune that was left by the father?

XXXIV. PROGRESSION,

Consisting of Two Parts,

ARITHMETICAL AND GEOMETRICAL.

ARITHMETICAL PROGRESSION

IS when a rank or series of numbers increase or decrease by a common difference, or by a continual adding or subtracting some equal numbers.

As $\left\{\begin{matrix}1, 2, 3, 4, 5, 6, 7, 8.\\ 8, 7, 6, 5, 4, 3, 2, 1.\end{matrix}\right\}$ Here the common difference is 1.

Or, 1, 3, 5, 7, 9, 11, 13. Here the common difference is 2.

Also 35, 30, 25, 20, 15, 10, 5. Here the common difference is 5.

1. In any series of numbers in arithmetical progression, when the number of terms are even, as 1, 3, 5, 7, 9, 11, or the like, the sum of the two extremes will be equal to the sum of any two means that are equally distant from the extremes:

Viz. 1, 3, 5, 7, 9, 11.

$1+11=5+7=3+9=12.$

2. When the number of terms are odd, as 2, 4, 6, 8, 10, the double of the middle figure or term will be equal to the the sum of the extremes, or any two means equally distant from the middle term:

Viz. 2, 4, 6, 8, 10.

$6\times 2=2+10=12$ and $6\times 2=4+8=12.$

In arithmetical progression there are five things to be observed, viz.

1. The first term.
2. The last term.
3. The number of terms.
4. The common excess, or difference.
5. The aggregate sum of all the terms.

Any three of which being given, the other two may be found.

PROPOSITION I.

When two extremes and the number of terms are given, to find the sum of all the series of terms.

RULE.

Multiply the sum of the two extremes into the number of terms, and divide the product by 2. The quotient will be the sum of all the series. Or multiply the sum of the two extremes by half the number of terms.

EXAMPLES.

(1) How many strokes do the clocks at Venice (which go on to 24 o'clock) strike in the compass of a natural day?

(2) How many strokes does the hammer of a clock strike in 12 hours?

(3) The length of a garden is 94 feet: now if eggs be laid along the pavement a foot asunder, and be fetched up singly to a basket, removed one foot from the first, how much ground does he traverse that does it?

(4) Suppose 100 stones were placed in a right line, a yard distant from one another, and the first stone were one yard from a basket; I demand how many miles he must travel that gathers them singly into the basket.

(5) A butcher bought 100 sheep, and gave for the first sheep 1*s.* and for the last 9*l.* 19*s.* I demand what he gave for the 100 sheep.

PROPOSITION II.

When the two extremes and number of terms are given, to find the common difference.

RULE.

The difference of the two extremes divided by the number of terms less a unity or 1, the quotient will be the common difference.

EXAMPLES.

(6) One had 20 children that differed alike in their ages: the youngest was 5 years old, the eldest 43. What was the difference of their ages, and the age of each?

(7) A running footman, for a wager, is to travel from

London to a certain place northwards in 19 days, and to go but 6 miles the first day, increasing every day's journey by an equal excess, so that the last day's journey may be 60 miles. I demand each day's journey, and what distance the place he goes to is from London.

(8) A debt is to be discharged at 10 different payments in Arithmetical Progression: the first payment is to be *5l.* and the last *30l.* What is the whole debt, and what must each payment be?

PROPOSITION III.

When the two extremes and the common difference are given, to find the number of terms.

RULE.

Divide the difference of the two extremes by the common excess or difference; add unity or 1 to the quotient, and the sum will be the number of terms.

EXAMPLES.

(9) A man being asked how many children he had, answered, that his youngest child was 5 years old, and the eldest 43, and that he had increased one in his family every two years. How many children had he?

(10) A person travelling from London northward, went 6 miles the first day, and increased every day's journey 3 miles, till at last he went 60 miles in one day. How many miles did he travel?

PROPOSITION IV.

When the last term, the common difference, and the number of terms are given, to find the first term.

RULE.

Multiply the number of terms, less unity or 1, by the common difference; the product subtracted from the last term leaves the first.

EXAMPLES.

(11) A man in 19 days went from London to a certain place in the country; every day's journey was greater than the preceding one by 3 miles: his last day's journey was 60 miles. What was the first?

(12) A person takes out of his pocket, at 10 different times, so many different numbers of guineas, every one exceeding the former by two; the last was 23. What was the first?

PROPOSITION V.

When the number of terms, common difference, and the sum of all the terms are given, to find the first term.

RULE.

Divide the sum of all the series by the number of terms, and from that quotient subtract half the product of the common difference multiplied by the number of terms less one, gives the first term.

EXAMPLES.

(13) A person is to receive 275*l.* at 10 different payments, each payment to exceed the former by 5*l.*: he is willing to bestow the first payment on any one that can tell what it is. What must the arithmetician have for his pains?

(14) Suppose it be 100 leagues between London and Edinburgh, two couriers set out from each place on the same road; that from London towards Edinburgh travelling every day two leagues more than the day before; that from Edinburgh to set off one day after the other, travelling every day three leagues more than the preceding one, and that they meet exactly half way, the first at the end of five days, and the other at the end of four: how many leagues did each travel per day?

PROPOSITION VI.

When the first term, number of terms, and the common difference are given, to find the last term.

RULE.

Subtract the common difference from the product of the number of terms, multiplied by the common difference: the remainder, added to the first term, will give the last.

EXAMPLES.

(15) What is the last term of an arithmetical progression, beginning at 6, and continuing by the increase of 3 to 19 places?

(16) What is the last term of an arithmetical progression, beginning at 1, and continuing by the increase of 2 to 100 places?

PROPOSITION VII.

The first term, common difference, and number of terms being given, to find the sum of all the series.

RULE.

From the product of the number of terms in the common difference, subtract the common difference; and to the remainder add the double of the first term: half the product of that sum multiplied by the number of terms, gives the sum of all the series.

EXAMPLE.

(17) A gentleman bargains with a bricklayer to sink him a well 30 yards deep, upon these terms, viz. to pay him three shillings for the first yard, five for the second, seven for the third, &c. raising two shillings for every yard. What will be due to the bricklayer for completing the same?

PROPOSITION VIII.

The first term, the number of terms, and sum of all the terms being given, to find the common difference.

RULE.

Divide the double sum of all the series by the number of terms, and from the quotient subtract double the first

term; divide the remainder by the number of terms lessened by unity; the quotient will be the common difference.

EXAMPLES.

(18) A gentleman travelled 200 miles in eight days, and every day travelled equally farther than the preceding day; it is known that the first day he travelled six miles. How many miles did he travel each of the other days?

(19) A person travelled from London to York, being 200 miles, in 9 days, and every day travelled equally farther than the preceding day; it is known that the first day he travelled 4 miles. How many miles did he travel each of the other days?

PROPOSITION IX.

When one person or thing moves with an equal, and another the same way, by a progressive motion, to find what time the first will be overtaken.

RULE.

To double the space gone each day by the pursued, add the common difference of the pursuer's day's journey; from that sum subtract double the space he travelled the first day, and divide the remainder by the common difference: the quotient will give the number of days in which the pursued will be overtaken by the pursuer.

EXAMPLES.

(20) A noted highwayman having committed a robbery, not suspecting a pursuit, fled northward at the rate of nine leagues a day; one of Sir John Fielding's men followed him in a progressive motion, only three leagues the first day, five the second, seven the third, and so on, increasing every day's journey two leagues. In how many days was the highwayman overtaken?

(21) Y. Z. made the following bet for 1000 guineas, to be decided the Monday, Tuesday, and Wednesday, in

Whitsun-week, on Barnham Downs, between the hours of eight in the morning and eight at night. The proposer has 10 choice cricketers in full exercise, who, on this occasion, are to be distinguished by the first 10 letters of the alphabet. These are to run and gather up, and carry singly 1000 eggs, laid in a right line, just two yards asunder, putting them gently into a basket placed just a fathom behind the first. They are to work one at a time, in the following order: A. is to fetch up the first 10 eggs, B. the second, C. the third 10, and so forward to K. whose turn it will be to fetch up the 1000th egg. After which A. sets out again for the next 10; B. takes the next, and so forward alternately, till K. shall have carried up the 1000th egg, at 100 eggs per man. The men are to have 300*l.* for their three days' work, if they do it; and it is to be distributed in proportion to the ground each man shall in his course have gone over. Required, first, how many miles each person will have run? secondly, what part of the 300*l.* will come to his share? thirdly, whether, if the men had been posted at proper places, they had not better have run from London to York twice, and back, in the time, taking the distance at 180 miles?

XXXV. GEOMETRICAL PROGRESSION

IS when any rank or series of numbers increase by one common multiplier, or decrease by one common divisor.

As 2. 4. 8. 16. 32. 64. Here the common multiplier or ratio is 2.

Also 729. 243. 81. 27. 9. 3. Here the common divisor or ratio is 3.

In any series of numbers in geometrical progression, the product of the two extremes is equal to the product of any two means that are equally distant from the extremes.

As 3. 9. 27. 81. 243. 729

Here $3 \times 729 = 27 \times 81 = 9 \times 243 = 2187$.

When the number of terms are odd, the middle term multiplied into itself will be equal to the product of the two extremes, or any two means equally distant from the said mean or middle term.

As 3. 6. 12. 24. 48.

$12 \times 12 = 6 \times 24 = 48 \times 3 = 144.$

In geometrical progression the same five things are to be observed as in arithmetical progression, viz.

1. The first term.
2. The last term.
3. The number of terms.
4. The ratio.
5. The sum of the terms.

Any three of these being known, the rest may be found.

If to any series of numbers in geometrical proportion, when the first term is not a unit, or the same as ratio, but not a unit, there be assigned a series of numbers in arithmetical progression, beginning with a unit or 1, and whose common difference is 1, called indices or exponents.

Thus $\begin{cases} 1.\ 2.\ 3.\ 4.\ 5.\ 6.\ 7. & \text{Indices.} \\ 2.\ 4.\ 8.\ 16.\ 32.\ 64.\ 128. & \text{Number in geometrical progression.} \end{cases}$

The addition or subtraction of the indices, or numbers in arithmetical progression, directly corresponds with the product or quotient of their respective terms or series in geometrical progression.

That is $\begin{cases} \text{As } 3+6=9. \\ \text{So } 8 \times 64 = 512 \text{ the 9th term in } \div \end{cases}$

Again $\begin{cases} \text{As } 6+6=12. \\ \text{So } 64 \times 64 = 4096 \text{ the 12th term in } \div \end{cases}$

Or $\begin{cases} \text{As } 6-3=3. \\ \text{So } 64 \div 8 = 8. \end{cases}$

Or $\begin{cases} \text{As } 7-2=5. \\ \text{So } 128 \div 4 = 32, \text{ \&c.} \end{cases}$

But if the series begin with unity, or 1, the indices must begin with a cipher.

Thus $\begin{cases} 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7, \text{ \&c.} & \text{Indices.} \\ 1,\ 2,\ 4,\ 8,\ 16,\ 32,\ 64,\ 128. \end{cases}$

Now by these indices, and a few of the first terms, the last term, or any distant one, may be speedily found, without producing the whole series.

PROPOSITION I.

When the first term is unity, the ratio and number of terms being known, to find the last or any remote term.

RULE.

Find a few of the leading terms, over which place their indices, as before directed; then find what figures of the indices, which added together will give the index of the term wanted: multiply the number standing under such indices into each other, and the last product will be the term required.

Note.—When the indices begin with a cipher, the sum of the indices made choice of must be always one less than the number of terms given in question, as 1 in the indices stands over the second term.

EXAMPLES.

(1) A boy agrees, for 16 oranges, to pay only the price of the last, reckoning a farthing for the first, and a halfpenny for the second, &c. doubling the price to the last. How much did he give for them?

(2) A man bought a horse, and by agreement was to give what the last nail would come to, at a farthing for the first nail, two for the second, four for the third, &c. There were 4 shoes, and 9 nails in each shoe. I demand the price of the horse.

PROPOSITION II.

In any series, not proceeding from unity, the ratio and first term being given, to find any remote term, without producing all the intermediate terms.

RULE.

Proceed as in the last proposition, only observe to divide every product by the first term, and the quotient will be the term required.

EXAMPLES.

(3) A person dying, left 11 children, to whom, and to his executor, he bequeathed in the manner following, viz. To his executor, for seeing his will performed, 10*l.*; the youngest child to have 30*l.*; and so on, every child to exceed the next younger in triple proportion. What will be the share of the eldest?

(4) A nobleman dying, left 10 sons, to whom he left a certain sum of money to be divided among them, viz. The youngest son to have 500*l.* the second to have as much and half as much, and so on, every one to exceed the next youngest in the same ratio of $1\frac{1}{2}$. What is the share of the eldest?

PROPOSITION III.

When the first term, ratio, and number of terms are given, to find the sum of all the terms.

RULE.

Find the last term as before, from which take the first; divide the remainder by the ratio, less one, and to that quotient add the last term; which gives the sum required.

EXAMPLES.

(5) On New-year's day a gentleman married, and received of his father-in-law a guinea, on condition that he was to have a present on the first day of every month for the first year, which should be double still to what he had the month before. What was the young lady's portion?

(6) One, at a country fair, had a mind to a string of 20 fine horses: but not caring to take them at 20 guineas per head, the jockey consented that he should, if he thought good, pay but a single farthing for the first, doubling it only to the 19th, and he would give the 20th into the bargain. This being presently accepted, how were they sold per head?

(7) A laceman, well versed in numbers, agreed with a gentleman to sell him 20 yards of rich gold brocaded lace, for 2 pins the first yard, 6 for the second, 18 for the third, and so on in triple proportion. I demand how much the lace produced. The pins afterwards sold at a farthing per 100. I demand whether the laceman gained or lost by the sale thereof, supposing the said lace to have been bought at 8*l.* 1*s.* 8*d.* per yard.

(8) A servant agreed with a master unskilled in numbers to serve him 11 years without any other reward for his

service but the produce of a wheat corn for the first year, and that product to be sown the second year, and so on from year to year, until the end of the time, allowing the increase to be but tenfold proportion. I demand what the 11 years' service came to, supposing the sum of the whole produce to be sold at 4s. per bushel.

Note.—7680 wheat corns, round and dry out of the middle of the ear, are computed to fill a statute pint.

PROPOSITION IV.

Of any decreasing series in ∺ whose last term is a cipher, to find the sum of those series.

RULE.

Divide the square of the first term by the difference between the said first term and the second term in the series; the quotient will be the sum of the series.

EXAMPLES.

(9) A great ship pursues a little one, steering the same way, at the distance of four leagues from it, and sails twice as fast as the small ship. It is asked how far the great ship must sail before it overtakes the lesser.

(10) Suppose a ball to be put in motion by a force which drives it 12 miles the first hour, 10 the second, and so on, continually decreasing in proportion of 12 to 10, to infinity. What space would it move through?

XXXVI. PERMUTATION,

OR,

VARIATION,

IS the changing or varying the order of things, in respect of their places.

RULE.

Multiply all the given terms in a series of Arithmetical Progressionals continually, whose first term or common

difference is unity or 1, and the last term the number of things proposed to be varied together: and the product will be the number of changes or variations required.

EXAMPLES.

(1) Six gentlemen that were travelling, met together by chance at a certain inn upon the road, where they were so pleased with their landlord, and each other's company, that in a frolic they made a contract to stay at that place so long as they, together with their landlord, could sit every day in a different order or position at dinner. Quere, the time they staid.

(2) I demand the number of changes that may be rung on 12 bells. Also, in what time they may all be rung, allowing 3 seconds to every round, and 365 days 6 hours to the year.

(3) An accomptant told a gentleman, who had constantly 8 persons at his table, that he would gladly make a ninth, and was willing to give 20 guineas for his board, so long as he could place the said company at dinner differently from any one day before. This being accepted, what did his entertainment cost him per year?

END OF BOOK II.

PART II.

XXXVII. VULGAR FRACTIONS.

A FRACTION is a part or parts of something considered as a unit or integer, and consists of two parts or quantities, one written over the other, with a line between them, as $\frac{1}{4}$, $\frac{3}{5}$, $\frac{15}{28}$, &c.

The number placed below the line is called the denominator of the fraction, because it denominates or shows how many parts the unit is broken or divided into; and the number above the line is called the numerator, because it enumerates or shows how many of those parts are contained in the fraction.

A vulgar fraction is either proper, improper, compound, or mixed.

A proper fraction is such whose numerator is less than the denominator, as $\frac{2}{5}$, $\frac{7}{8}$, $\frac{23}{36}$, $\frac{161}{215}$, &c.

An improper fraction is when the numerator is equal to, or greater than, its denominator, as $\frac{2}{2}$, $\frac{18}{12}$, $\frac{247}{42}$, &c.

A compound fraction is the fraction of a fraction, and known by the word *of*, as $\frac{3}{7}$ of $\frac{7}{8}$ of $\frac{4}{5}$, &c.

A mixed number is composed of a whole number and a fraction, as $4\frac{1}{4}$, $12\frac{7}{8}$, $142\frac{11}{19}$, &c.

XXXVIII. REDUCTION OF VULGAR FRACTIONS.

Case 1. To reduce a vulgar fraction to its lowest terms.

RULE.

Divide the greater term by the less, and that divisor by the remainder following, till nothing remain: then by the last

remainder divide both parts of the fraction, and the quotients will give the fraction required. If the remainder be 1, the fraction is already in its least terms.

EXAMPLES.

(1) Reduce $\frac{336}{408}$ to its lowest terms.
(2) Reduce $\frac{2832}{2848}$ to its lowest terms.
(3) Reduce $\frac{144}{560}$ to its lowest terms.
(4) Reduce $\frac{192}{336}$ to its lowest terms.
(5) Reduce $\frac{1476}{1938}$ to its lowest terms.

1. When the numerator and denominator each of them end with ciphers, strike off an equal number of ciphers in both, and the remaining figures will be a fraction of the same value, which reduce to its lowest terms.

EXAMPLES.

(6) Reduce $\frac{19000}{27000}$ to its lowest terms.
(7) Reduce $\frac{420}{3400}$ to its lowest terms.

2. When you discern any number will equally divide both numerator and denominator, you may abbreviate the fraction thereby.
(8) Reduce $\frac{24}{112}$, $\frac{96}{324}$, $\frac{60}{144}$, and $\frac{120}{360}$, to their lowest terms.

Case 2. To reduce a compound fraction to a single one.

RULE.

Multiply all the numerators together for a new numerator, and the denominators for a new denominator. Reduce the new fraction to its lowest terms by the last case. When it can be done, you may cancel the fractions, by dividing the numerator and denominator of any two terms by the same number, and use the quotient instead thereof.

EXAMPLES.

(9) Reduce $\frac{4}{5}$ of $\frac{3}{7}$ of $\frac{9}{10}$ to a single fraction.
(10) Reduce $\frac{2}{5}$ of $\frac{3}{4}$ of 4 to a single fraction.
(11) Reduce $\frac{3}{7}$ of $\frac{1}{3}$ of $\frac{7}{8}$ to a single fraction.

Case 3. To reduce whole or mixed numbers into an improper fraction.

RULE.

1. If the whole number have no assigned denominator, a unity subscribed underneath must be the denominator.

2. If the whole number have an assigned denominator, multiply the whole number by the assigned denominator, and the product will be the numerator to the assigned denominator.

3. If the whole number have a fraction annexed, multiply the whole number by the denominator of the fraction, and to the product add the numerator for a new numerator, which place over the denominator.

EXAMPLES.

(12) Reduce 12, 27, and 176, to fractions.
(13) Reduce 27 into a fraction, whose denominator shall be 12.
(14) Reduce $4\frac{2}{7}$ to an improper fraction.
(15) Reduce $16\frac{1}{4}$ to an improper fraction.
(16) Reduce $142\frac{17}{23}$ to an improper fraction.
(17) Reduce $146\frac{21}{37}$ to an improper fraction.

Case 4. To reduce an improper fraction into its equivalent or proper terms.

RULE.

Divide the numerator by the denominator: the quotient gives the whole number, and under the remainder (if any) subscribe the denominator.

EXAMPLES.

(18) Reduce $\frac{20}{7}$ to its proper terms.
(19) Reduce $\frac{13}{8}$ to its proper terms.
(20) Reduce $\frac{283}{21}$ to its proper terms.
(21) Reduce $\frac{423}{37}$ to its proper terms.

Case 5. To reduce fractions of different denominations to fractions of equal value, that shall have one common denominator.

RULE.

Multiply each numerator, taken separately into all the

denominators but its own, and the products will be the new numerators: then multiply all the denominators into one another for a common denominator.

EXAMPLES.

(22) Reduce $\frac{3}{4}$, $\frac{11}{12}$, and $\frac{7}{10}$ to a common denominator.
(23) Reduce $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$ of $\frac{7}{8}$, to a common denominator.
(24) Reduce $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$, to a common denominator.
(25) Reduce $\frac{5}{6}$, $\frac{7}{10}$, $\frac{3}{4}$, and $\frac{3}{4}$ of 3, to a common denominator.

Case 6. To reduce fractions of one denomination to another, retaining the same value.

RULE.

1. If the fraction given is to be brought from a less to a greater denomination, multiply the denominator by the parts contained in the several denominations between it and that you would reduce it to, for a new denominator; which, placed under the given numerator, will give the new fraction, which reduce to its lowest terms.

2. If the fraction given is to be brought from a greater to a less denomination, then multiply the numerator in the same manner as you did before the denominator, and place over the given denominator, and it will give the new fraction, which also reduce to its lowest terms.

EXAMPLES.

(26) Reduce $\frac{3}{4}$ of a shilling to the fraction of a guinea.
(27) Reduce $\frac{1}{620}$ of a £. to the fraction of a penny.
(28) Reduce $\frac{3}{4}$ of a farthing to the fraction of a moidore.
(29) Reduce $\frac{5}{7}$ of a penny to the fraction of a £.
(30) Reduce $\frac{1}{1344}$ of a guinea to the fraction of a farthing.
(31) Reduce $\frac{5}{6}$ of a dwt. to the fraction of a lb. Troy.
(32) Reduce $\frac{3}{448}$ of a cwt. to the fraction of a lb. avoirdupois.
(33) Reduce $\frac{3}{4}$ of a drachm to the fraction of a cwt.
(34) Reduce $\frac{5}{144}$ of a lb. Troy to the fraction of a dwt.
(35) Reduce $\frac{7}{9}$ of a league to the fraction of a pole.
(36) Reduce $\frac{7}{45}$ of a yard to the fraction of a nail.

(37) Reduce $\frac{11}{12}$ of a gallon of wine to the fraction of a hogshead.
(38) Reduce $\frac{7}{9}$ of a hhd. of ale to the fraction of a pint.
(39) Reduce $\frac{7}{144}$ of a chaldron to the fraction of a bushel.
(40) Reduce $\frac{3}{5}$ of a gallon to the fraction of a chaldron.
(41) Reduce $\frac{7}{142}$ of a week to the fraction of a second.
(42) Reduce $\frac{10}{11}$ of a minute to the fraction of a day.

Case 7. To find the proper quantity or value of a fraction in money, weights, measures, &c.

RULE.

Multiply the numerator of the given fraction by the parts contained in the integer to which it belongs; then divide that product by the denominator; and, if any thing remain, reduce it to the next denomination less, and divide again by the denominator; thus proceed to the least denomination, and the quotients placed in their order will be the answer.

EXAMPLES.

(43) Reduce $\frac{367}{1296}$ of a moidore to its proper quantity.
(44) Reduce $\frac{5}{12}$ of a guinea to its proper quantity.
(45) Reduce $\frac{5}{8}$ of a shilling to its proper quantity.
(46) Reduce $\frac{4}{5}$ of a three-pound-twelve to its proper quantity.
(47) What is the value of $\frac{5}{7}$ of a £.?
(48) Reduce $\frac{411}{740}$ of a lb. Troy to its proper quantity.
(49) What is the value of $\frac{7}{9}$ of a cwt.?
(50) Reduce $\frac{63}{124}$ of a ton to its proper quantity.
(51) What is the value of $\frac{7}{8}$ of a mile?
(52) Reduce $\frac{5}{8}$ of an ell English to its proper quantity.
(53) Reduce $\frac{7}{8}$ of an acre to its proper quantity.
(54) What is the value of $\frac{11}{212}$ of a hhd. of wine?
(55) Reduce $\frac{7}{9}$ of a barrel of beer to its proper quantity.
(56) Reduce $\frac{25}{72}$ of a chaldron of coals to its proper quantity.
(57) What is the value of $\frac{7}{8}$ of a month?
(58) Reduce $\frac{5}{7}$ of a day to its proper quantity.

Case 8. To reduce money, weights, or measures, into fractions.

RULE.

Reduce the given quantity to the lowest name mentioned, for a numerator; under which put the number of those parts contained in a unit of the integer for a denominator, then reduce the fraction to its lowest terms.

EXAMPLES.

(59) Reduce 6*s.* 8½*d.* to the fraction of a pound.
(60) Reduce 4½*d.* to the fraction of a shilling.
(61) Reduce 2*l.* 17*s.* 7½*d.* to the fraction of a three-pound-twelve.
(62) Reduce 8*s.* 2*d.* to the fraction of a guinea.
(63) Reduce 6 oz. 17 dwts. 21 grs. to the fraction of a lb. Troy.
(64) Reduce 3 qrs. 3 lb. 1 oz. 12⅕ drs. to the fraction of a cwt.
(65) Reduce 10 cwt. 18 lb. 1 $\frac{1}{31}$ oz. to the fraction of a ton.
(66) Reduce 3¼ qrs. to the fraction of an ell English.
(67) Reduce 2 f. 6 in. to the fraction of a yard.
(68) Reduce 4 fur. 32 p. to the fraction of a mile.
(69) Reduce 3 r. 2 p. to the fraction of an acre.
(70) Reduce 42 gal. of wine to the fraction of a hhd.
(71) Reduce 28 gal. of beer to the fraction of a barrel.
(72) Reduce 14 bu. 2 p. to the fraction of a chaldron.
(73) Reduce 1 w. 3 d. 12 h. to the fraction of a month.
(74) Reduce 243 d. 8 h. to the fraction of a year, allowing 365 days to the year.

XXXIX. ADDITION of VULGAR FRACTIONS.

Case 1. To add fractions together having different denominators.

RULE.

1. Reduce the given fractions to a common denominator by Case V. in Reduction.

2. Add all the numerators together for a new numerator, under which subscribe the common denominator. And, if it be an improper fraction, reduce it to its proper terms by Case IV. and you have the sum of all the fractions.

EXAMPLES.

(1) Add $\frac{3}{7}$, $\frac{1}{4}$, and $\frac{5}{6}$ together.
(2) Add $\frac{1}{8}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$ together.
(3) Add $\frac{2}{3}$ of $\frac{4}{9}$, and $\frac{13}{15}$, together.
(4) Add $\frac{3}{7}$ of $\frac{5}{6}$, to $\frac{5}{9}$ of $\frac{3}{4}$.
(5) Add $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$, of $\frac{5}{6}$, into one sum.

Case 2. To add mixed numbers.

RULE.

Reduce the fractions to a common denominator, and add them together, as before directed, and annex their sum to the sum of the integers.

EXAMPLES.

(6) Add $4\frac{1}{2}$ and $17\frac{3}{4}$ together.
(7) Add $7\frac{2}{3}$, $9\frac{4}{5}$, and $6\frac{7}{10}$, together.
(8) Add $8\frac{1}{2}$, $9\frac{2}{3}$, $10\frac{3}{4}$, $11\frac{4}{5}$, and $12\frac{5}{6}$, into one sum.

Case 3. When the given fractions are of several denominations.

RULE.

Reduce them to one denomination, by Case VIII. then add them together as before directed; or you may reduce them to their proper quantities, by Case IX. and add them together.

EXAMPLES.

(9) Add $\frac{201}{400}$ of a £. to $\frac{1}{8}$ of a shilling.
(10) Add $\frac{1}{4}$ of a lb. Troy to $\frac{3}{8}$ of an oz.
(11) Add $\frac{5}{8}$ of a cwt. to $\frac{1}{3}$ of a lb.
(12) Add $\frac{3}{4}$ of a yard, to $\frac{2}{5}$ of an ell English.
(13) To $\frac{1}{6}$ of a mile, add $\frac{2}{3}$ of a yard.
(14) Add $\frac{3}{8}$ of a chaldron, to $\frac{1}{2}$ of a peck.
(15) To $\frac{3}{4}$ of a week, add $\frac{5}{6}$ of a month.
(16) Add $\frac{3}{4}$ of an hour, to $\frac{5}{8}$ of a week.
(17) Add $\frac{2}{5}$ of $12l. + 4\frac{3}{7}l. + \frac{1}{5}$ of $\frac{9}{10}$ of a £. $+ \frac{2}{3}$ of $\frac{5}{6}$ of a shilling, into one sum.

XL. SUBTRACTION of VULGAR FRACTIONS.

RULE.

1. PREPARE the fractions as before directed in Addition.

2. Subtract one numerator from the other, and their difference will be a new numerator, under which subscribe the common denominator.

EXAMPLES.

(1) It is required to subtract $\frac{2}{9}$ from $\frac{11}{12}$.
(2) From $\frac{19}{64}$ take $\frac{2}{15}$.
(3) From $\frac{14}{17}$ take $\frac{7}{11}$.
(4) From $\frac{3}{4}$ of $\frac{7}{8}$ take $\frac{2}{5}$.
(5) From $\frac{5}{6}$ of $\frac{7}{10}$ take $\frac{2}{3}$ of $\frac{1}{2}$.
(6) From $10\frac{3}{4}$ take $6\frac{5}{6}$.
(7) From $17\frac{11}{12}$ take $16\frac{6}{7}$.
(8) From 12 take $\frac{5}{6}$ of $\frac{7}{8}$ of $\frac{3}{4}$.
(9) From $\frac{2}{3}$ of a shilling take $\frac{3}{4}$ of a penny.
(10) From $\frac{1}{4}$ of a cwt. take $\frac{3}{8}$ of an oz.
(11) From $\frac{3}{4}$ of a yard take $\frac{3}{8}$ of an inch.
(12) From $\frac{3}{8}$ of a chaldron take $\frac{3}{4}$ of a peck.
(13) From $\frac{5}{6}$ of a day take $\frac{3}{4}$ of an hour.

XLI. MULTIPLICATION of VULGAR FRACTIONS.

RULE.

1. REDUCE compound fractions to simple ones, per Case II.; bring mixed numbers into improper fractions, per Case III.

2. Multiply the numerators together for a new numerator; and the new denominators for a new denominator; or, if the fractions will cancel, do it as in Case II.

EXAMPLES.

(1) Mul. $\frac{6}{11}$ by $\frac{7}{11}$.
(2) Mul. $12\frac{1}{2}$ by 6.
(3) Mul. $17\frac{3}{5}$ by $\frac{7}{8}$.
(4) Mul. $2\frac{1}{4}$ by $\frac{3}{4}$ of $\frac{7}{8}$.
(5) Mul. $12\frac{1}{4}$ by $\frac{7}{8}$ of 12.
(6) Mul. $\frac{3}{4}$ of $\frac{10}{11}$ by 6.
(7) Mul. $\frac{7}{8}$ of $\frac{3}{4}$ by $\frac{2}{5}$ of $\frac{5}{7}$ of 14.
(8) Mul. $3\frac{3}{5}$ by $\frac{5}{7}$, and this product again by $\frac{3}{5}$ of $\frac{3}{4}$.

XLII. DIVISION of VULGAR FRACTIONS.

RULE.

1. PREPARE the fractions as before directed in multiplication.

2. Multiply the numerator of the dividend into the denominator of the dividing fraction for a new numerator, and multiply the other numerator and denominator together for a new denominator; or invert the divisor, and then proceed as in the last rule.

EXAMPLES.

(1) Divide $\frac{6}{7}$ by $\frac{3}{5}$.
(2) Divide $\frac{2}{11}$ by $\frac{2}{3}$ of $\frac{7}{8}$.
(3) Divide $12\frac{1}{2}$ by $17\frac{2}{3}$.
(4) Divide $12\frac{3}{4}$ by $3\frac{7}{8}$.
(5) Divide $\frac{7}{8}$ of $\frac{3}{4}$ by $\frac{2}{5}$ of 12.
(6) Divide $7\frac{3}{7}$ by 9.
(7) Divide 96 by 146.
(8) Divide $14\frac{3}{7}$ by $\frac{3}{4}$ of 12.
(9) Divide $142\frac{7}{12}$ by $12\frac{3}{5}$.
(10) Divide $\frac{7}{8}$ of 6 by $\frac{3}{4}$ of $\frac{6}{7}$ of $\frac{11}{12}$.

XLIII. THE RULE of THREE DIRECT,

In Vulgar Fractions.

THERE are two methods to perform this rule: the second of which is the easiest and most expeditious.

RULE.

1. Prepare the fractions, if required, as directed in multiplication; then proceed as in Sect. 12.—Or,

2. Having reduced the fractions, and stated the question, as before directed;

3. Multiply the donominator of your first number into the numerators of the second and third, for a new numerator; then multiply the numerator of the first number into the denominator of the second and third, for a new denominator, and place it under the new numerator, for an answer, which reduce to its proper quantity: or invert the first term, and then proceed as in Case 2, Sect 38; or as in Multiplication.

EXAMPLES.

(1) If $2\frac{2}{5}$ yards of silk cost $3\frac{3}{4}$*l.* what will $4\frac{4}{5}$ yards cost at the same rate?

(2) If $\frac{3}{4}$ of a lb. cost 5*s.* 6*d.* what will $42\frac{3}{8}$ lb. of the same cost?

(3) Suppose I give 14*s.* 8*d.* for $\frac{7}{8}$ cwt. what must be given for 8 cwt. of the same at that rate?

(4) A merchant makes an assurance upon a ship and cargo bound to a certain port, value 2700*l.* 10*s.* and agrees to pay 10 guineas per cent. To what comes the premium or charges of the assurance?

(5) How much South-sea stock, at $112\frac{5}{8}$*l.* per cent. will 1270*l.* purchase?

(6) A mercer bought $4\frac{3}{4}$ pieces of silk, each piece containing $22\frac{3}{8}$ yards, and was to give 8*s.* 9*d.* per yard. I demand the value of the whole.

(7) If I give 100*l.* 10*s.* 6*d.* for 12 pieces of Holland, at the rate of 5*s.* $6\frac{3}{4}$*d.* per ell Flemish; I demand how many ells English each piece contained.

XLIV. THE RULE of THREE INVERSE,

In Vulgar Fractions.

AS I observed in the Rule of Three Direct, of there being two methods of performing it, so likewise in this.

RULE.

1. Prepare the fractions as before directed, and then proceed as in Sect. 13.—Or,

2. Multiply the denominator of the third number into the numerator of the first and second for a new numerator; then multiply the numerator of the third number into the denominator of the first and second, for a denominator, which place under the numerator for an answer, and find the proper quantity as before; or invert the last term, and proceed as in the last rule.

EXAMPLES.

(1) A. lends B. $25\frac{1}{3}$*l.* for $6\frac{3}{4}$ months. How long ought B. to let A. have $10\frac{3}{4}$*l.* to requite his kindness?

(2) If 4 men can do a piece of work in $12\frac{3}{4}$ hours, in how many hours will 12 men do the same?

(3) If the penny loaf weigh $12\frac{2}{3}$ oz. when the bushel of wheat is sold for 5*s.* what is the load worth when the penny loaf weighs $8\frac{3}{4}$ oz.?

(4) Suppose A. lends to B. $100\frac{3}{7}$*l.* for $6\frac{2}{3}$ months, what sum must B. lend A. for $8\frac{5}{8}$ years to requite him?

(5) How many yards of cloth, at 8*s.* 6*d.* per yard, must be given for $26\frac{5}{6}$ yards, at 5*s.* 7*d.* per yard?

XLV. THE DOUBLE RULE of THREE,

In Vulgar Fractions.

PREPARE the numbers, as before directed, and then proceed as in page 74.

EXAMPLES.

(1) What principal, put to interest, will gain 4*l.* 15*s.* in 9 months, at 6*l.* per cent. per annum?

(2) Suppose 12 students spend 14*l.* 6*s.* 8*d.* in 16 days, how much will 18 students spend in 34 days?

(3) If the carriage of 40 cwt. 30 miles, cost 16*l.* 13*s.* 4*d.* what weight may I have carried 80 miles for 6*l.* 17*s.* 6*d.* at the same rate?

(4) Six men with their wives, upon calculation, found that their expenses for three months past (allowing 30 days to one month) amounted to 26*l.* 19*s.* 4*d.* I demand in what time 14*d.* 15*s.* may be spent by 36 men in the like proportion.

(5) If 30 men can perform a piece of work in eleven days, how many will accomplish another four times as large in one fifth of the time?

(6) Agreed for the carriage of $2\frac{1}{2}$ tons of goods, 3 miles wanting $\frac{1}{10}$, for $\frac{1}{8}$ of $\frac{3}{5}$ of a guinea. What was that per cwt. for a mile?

QUESTIONS *for Exercise in* FRACTIONS.

(1) FOUR figures of nine may be so placed and disposed of as to denote and read for 100, neither more nor less. Pray how is that to be done?

(2) What number is that, to which if $\frac{3}{10}$ of $\frac{18}{7}$ of $\frac{141}{213}$ be added, the total will be 1?

(3) What number is that, from which if you deduct the $\frac{1}{25}$ of $\frac{7}{8}$, and to the remainder add $\frac{1}{20}$ of $\frac{47}{19}$, the sum will be 3?

(4) What number is that, to which if you add $\frac{1}{11}$ of 12, more $\frac{7}{20}$ of 27, and from the total subtract $\frac{1}{3}$ of $7\frac{1}{2}$ less $\frac{29}{36}$ of $1\frac{1}{4}$, the remainder shall be 8?

(5) There is a number, which, if multiplied by $\frac{3}{4}$ of $\frac{7}{8}$ of $2\frac{3}{4}$, will produce no more than 1. What is the cube of that number?

(6) There is a number, which, if divided by $\frac{2}{3}$ of $\frac{3}{16}$ will quote $9\frac{28}{37}$. What is the square of that number?

(7) If $\frac{3}{7}$ of $\frac{4}{5}$ of $\frac{7}{8}$ of a ship be worth $\frac{1}{9}$ of $\frac{6}{7}$ of $\frac{11}{13}$ of the cargo, value at 1200*l.* what did both ship and cargo stand the owners in?

(8) A person was possessed of a $\frac{3}{5}$ share of a copper mine, and sold $\frac{3}{4}$ of his interest therein for 1710*l.* what was the reputed value of the whole property at the same rate?

(9) A father devised $\frac{34}{83}$ of his estate to one of his sons, and $\frac{44}{83}$ of the residue to another, and the surplus to his relict, for her life; the children's legacies were found to be 257*l.* 3*s.* 4*d.* different. Pray what money did he leave the widow the use of?

(10) A person, making his will, gave to one child $\frac{19}{30}$ of his estate, to another $\frac{11}{19}$; and when these legacies came to be paid, one turned out 540*l.* 10*s.* more than the other. What did the testator die worth?

(11) A lad, having 4000 nuts, in his return home was met by Mad Tom, who took from him $\frac{5}{8}$ of $\frac{2}{3}$ of his whole stock. Raving Ned afterwards forced $\frac{3}{5}$ of $\frac{5}{8}$ of the remainder from him; unluckily Positive Jack found him, and required $\frac{7}{10}$ of $\frac{17}{20}$ of what he had left. Smiling Dolly was, by promise, to have $\frac{3}{4}$ of a quarter of what nuts he brought home. How many then had the boy left?

(12) A younger brother received 2200*l.* which was just $\frac{5}{12}$ of his eldest brother's fortune; and 3 and $\frac{1}{8}$ times the elder's money was $\frac{1}{2}$ as much again as the father was worth. What was that?

(13) In distress at sea, they threw out 17 hhds. of sugar, worth 34*l.* per hhd.; the worth of which came to but $\frac{4}{7}$ of the indigo they cast overboard: besides which, they threw out 13 iron guns worth 18*l.* 10*s.* a-piece; the value of all amounted to $\frac{3}{7}$ of $\frac{9}{13}$ of that of the ship and lading. What part of the value came into port?

(14) If A. having $\frac{7}{8}$ of $\frac{3}{5}$ of the half of a trading sloop and cargo, worth 16131 $\frac{7}{10}$*l.*, sells his brother B. $\frac{3}{5}$ of $\frac{4}{5}$ of his interest therein at prime cost; what did it cost the brother, and what did his cousin P. pay at the same time for $\frac{9}{11}$ of the remainder?

(15) X. Y. and Z. can, working together, complete a staircase in 12 days; Z. is able to do it alone in 24 days, and X. in 34. In what time then could Y. get it done himself?

(16) A father dying left his son a fortune, $\frac{3}{16}$ of which he ran through in six months; $\frac{4}{5}$ of the remainder lasted him a twelvemonth longer, at which time he had barely 348*l.* left. What did his father bequeath him?

(17) Kitty told her brother George that though her fortune on her marriage took 19312*l.* out of the family, it was but $\frac{3}{5}$ of two years' rent. Pray what was it?

(18) A merry young fellow in a short time ran through $\frac{1}{3}$ of his fortune; by advice of his friends he then gave 2200*l.* for an exempt's place in the guards; his

profusion continued till he had no more than 880 guineas left, which he found by computation was just $\frac{1}{20}$ part of his money after the commission was bought. What was his fortune at first?

(19) A person dying, left his wife with child, and making his will, ordered, that if she went with a son, $\frac{2}{3}$ of the estate should belong to him, and the remainder to his mother; and, if she went with a daughter, he appointed the mother $\frac{2}{3}$, and the girl $\frac{1}{3}$. But it happened that she was delivered both of a son and daughter; by which she lost in equity 2000*l.* more than if she had had only a girl. What would have been her dowry, had she only had a son?

(20) A cistern holds 103 gallons; and being brim-full, has two cocks to run off the water: by the first of which a three-gallon pail will be filled in 60 seconds, by the other in 75. In what time will this cistern be emptied through both these apertures together, supposing the efflux of the water all the same?

(21) A person having about him a certain number of crowns, said, if $\frac{1}{4}+\frac{1}{3}+\frac{1}{6}$ of what he had were added together, they would make just 45. How many crowns had he about him?

(22) A gentleman has an orchard of fruit-trees, one half of the trees bearing apples, one fourth pears, one sixth plums, and fifty of them bearing cherries. How many fruit-trees in all grow in the said orchard?

(23) A schoolmaster being asked how many scholars he had, answered, if I had as many, and $\frac{1}{2}$ as many, and $\frac{3}{4}$ as many, I should have 99. How many had he?

(24) In the year I wrote this, if to my age you add
$\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$ (thereof), with $\frac{4}{5}$ more,
The number 74 will then be had —
Ingenious youths, my age explore.

(25) A. in a scuffle, seized on $\frac{2}{3}$ of a parcel of sugar-plums; B. catched $\frac{3}{8}$ of it out of his hands, and C. laid hold on $\frac{3}{10}$ more; D. ran off with all A. had left, except $\frac{1}{7}$, which E. afterwards secured slily for himself; then A. and C. jointly set upon B., who, in the conflict, shed $\frac{1}{2}$ he had, which were equally picked up by D. and E. who laid *perdue*. B. then kicked down

C.'s hat, and they all scrambled anew for what it contained; of which A. got $\frac{1}{4}$, B. $\frac{1}{2}$, D. $\frac{3}{7}$. and C. and E. equal shares of what was left of that stock; D. then struck $\frac{3}{4}$ of what A. and B. last acquired out of their hands: they with difficulty recovered $\frac{1}{8}$ of it in equal shares again, but the other three carried off $\frac{1}{3}$ a-piece of the same. Upon this they called a truce, and agreed, that the $\frac{1}{3}$ of the whole left by A. at first should be equally divided among them. How much of the prize, after this distribution, remained with each of the competitors?

PART III.

XLVI. DECIMAL FRACTIONS.

A DECIMAL fraction is a fraction whose denominator is always unity or 1, with one or more ciphers. Thus, a unit may be imagined to be equally divided into 10 parts, and each of these into 10 more; so that, by a continual decimal subdivision, the unit may be supposed to be divided into 10, 100, 1000, and so on without end, all being equal parts, called tenth, hundredth, or thousandth parts of unit or 1.

In Decimal Fractions, the figures of the numerator only are expressed, the denominator being omitted, because it is always known to consist of a unit with so many ciphers as there are places in the numerator.

A decimal fraction is distinguished from an integer by a point or comma prefixed, thus, ,5 which stands for $\frac{5}{10}$, or $\frac{1}{2}$; ,75 for $\frac{75}{100}$ or $\frac{3}{4}$; ,2752 for $\frac{2752}{10000}$; and 12,010 for $12\frac{10}{1000}$ or $12\frac{1}{100}$, &c.

Ciphers at the right-hand of a decimal fraction alter not its value; for, ,5 or ,50 or ,5000 are each of them the same value, and are equal to $\frac{5}{10}$ or $\frac{1}{2}$; but ciphers at the left hand, in a decimal fraction, decrease the value in a tenfold proportion, for ,05 is $\frac{5}{100}$; also, ,0005 is $\frac{5}{10000}$, &c. all of which will plainly appear by the following

TABLE.

7	6	5	4	3	2	1	2	3	4	5	6	7	&c.
Millions.	C. of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.	Parts of Ten.	Parts of one Hundred.	Parts of one Thousand.	Parts of ten Thousand.	Parts of one C. Thousand.	Parts of a Million.	

By the above Table it also plainly appears, that as whole numbers increase towards the left hand by a tenfold proportion, so decimal parts decrease towards the right hand by the same proportion.

A finite decimal is that which ends at a certain number of places; but an infinite is that which no where ends.

A circulating or recurring decimal is that wherein one or more figures are continually repeated.

Thus 64,766666, &c. or 64,7$\dot{6}$, is called a single circulating or recurring decimal.

And 147,642642, &c. or 147,$\dot{6}4\dot{2}$, is called a compound recurring decimal.

Note.—In all operations, if the result consist of several nines, reject them, and make the next superior place a unit more. Thus, for 17,1999 write 17,2; and for 12,99 write 13, &c.

XLVII. ADDITION of DECIMALS.

Case 1. ADDITION and Subtraction in Decimals are performed after the same manner as Sect. 2, 3, of whole numbers: Care being taken that like parts be placed under one another; and from their sum or difference cut off so many decimal parts as there are the most in any of the given numbers.

EXAMPLES.

(1) What is the sum of ,0476, 21,476, ,0067, ,64, 17,6, and ,20764?

(2) Add ,427, 64,075, 27,6421, 10,8, ,0074, ,104, and ,046842, together.

(3) What is the sum of ,274, ,076, ,64762, ,0706, ,47, ,007, and 968,42?

Case 2. To add decimals, wherein there are single repetends.

RULE.

Repeat the circulating decimals till each line has an equal number of decimal places, and ends directly under each other, annexing a cipher or ciphers to the finite terms; then add as before; only increase the sum of the right hand row with as many units as it contains nines, and the figures in the sum under that place will be a repetend.

EXAMPLES.

(4) What is the sum of 47,674, 4,02642, 32,6, 6,14, and 27,0646?

(5) Add 11,4, 6,14274, 91,78, 37,671, and 146,476741.

(6) What is the sum of 14,276421, 7,4, 21,646, 9,27, and 31,1474?

Case 3. To add decimals, having compound repetends.

RULE.

Make the repetends similar and conterminous; then add as before, only increase the right-hand figure by as many units as are carried from the column of figures wherein all the repetends begin together; lastly, dash off for a repetend as many places as were so in the numbers added together.

EXAMPLES.

(1) What is the sum of 14,2471, 768,746, 7,064, and 26,0062?

(2) Add ,246, 3,67, 27,0427694, and 9,946, together.

XLVIII. SUBTRACTION of DECIMALS.

EXAMPLES.

(1) WHAT is the difference between 176, and 10,764?

(2) From 647 take ,00746.

(3) What is the difference between 74,6407, and 69,5?

Case 2. To subtract decimals that have a single repetend.

RULE.

Repeat the circulating decimals till the lines end together, as in addition; and if the repetend of the number to be subtracted be greater than the repetend of the number it is to be taken from, then the right-hand figure of the remainder must be less by unity than it would be; or instead of borrowing ten, as in whole numbers or finites, borrow in this place 9, the rest as usual, and the right-hand place or figure will be a repetend.

EXAMPLES.

(4) What is the difference between 41,7$\not{4}$, and 21,94648?

(5) From 24,1466, take 19,$\not{9}$.

(6) What is the difference between 16,1$\not{2}\not{6}$ and 4,194276$\not{4}$?

XLIX. MULTIPLICATION of DECIMALS.

Case 1. MULTIPLICATION of Decimals is also performed as in whole numbers, no regard being had to the decimals, as such, till the product is obtained; then observe the following

RULES.

1. Strike off so many figures from the right hand of the product, as there are decimal places in the multiplier and multiplicand.

2. But if there be not so many figures in the product, supply the deficiency by prefixing ciphers to the left hand, to make them equal.

3. If the number is to be multiplied by 10, 1000, &c. remove the separating point in the multiplicand so many places towards the right hand as there are ciphers in the multiplier.

EXAMPLES.

(1) Multiply ,17504, by 76. (2) Mul. 27,42, by 3,56.
(3) Mul. 8,04704, by ,2575 (4) Mul. 5745, by ,0675.

(5) Mul. ,4 by ,2. (6) Mul. ,047 by ,046.
(7) Mul. ,000476 by ,00078. (8) Mul. ,47 by ,0008.
(9) Mul. 1,076 by 1000. (10) Mul. ,42768 by 10000.

Case 2. When the product will contain more decimals than are necessary for the present purpose, the work may be contracted by the following

RULE.

Write down the multiplicand as usual, then write under it the multiplier inverted, with the unit place thereof under that place of the multiplicand, whose place you intend the product shall extend to; then multiply as usual by each figure of the multiplier, beginning with those of the multiplicand which stand over it, neglecting those to the right-hand, unless so far as to observe what would arise from multiplying the figures immediately, by carrying 1 from 5 to 15, 2 from 15 to 25, and from 25 to 35 carry 3, &c. which must be taken in at the beginning of each line. The first figure of each particular product must stand in a right line, underneath one another.

EXAMPLES.

(11) It is required to multiply 3,47678, by 27,6782, and to have only three places of decimals in the product.
(12) Multiply 47,689464 by 26,17694, retaining five decimals in the product.

Case 3. If the right-hand figure of the multiplicand be a circulate.

RULE.

Multiply the multiplicand as before, by every figure in the multiplier; observing to increase the right-hand of each resulting line by as many units as there are nines in the product of the first figure in that line; and the right-hand figure of each line will be a circulate; and before you add them together, make them all end at the same place, as shown in addition.

EXAMPLES.

(13) Let it be required to multiply 14764 by ,7.
(14) Multiply 42,642 by ,276.

Case 4. When the right-hand figure of the multiplier is a circulate.

RULE.

Multiply by the circulate as by a finite digit, then divide the product by 9; continue the quotient till the number of decimal places are equal to those in the product, which will give the true one; and proceed with the remainder of the multiplier, as in the whole numbers, pointing off for decimals, as in Case 1.

EXAMPLES.

(15) Multiply 46,2762 by $\frac{8}{9}$.

(16) Let it be required to multiply 261,276 by ,4$\frac{7}{9}$.

Case 5. When the multiplicand and multiplier are each a single circulate.

RULE.

The first line (or that produced by multiplying by the circulate in the multiplier) must be managed as in the last case, only the right-hand figure must be increased by as many units as there are nines in the product of the first figure of that line: the product of the rest must be managed as directed in Case 3.

EXAMPLES.

(17) Multiply 141,1$\frac{4}{9}$ by 8,47.

(18) Let it be required to multiply 24,607$\frac{7}{9}$ by ,49$\frac{6}{9}$.

Case 6. If the multiplicand be a compound repetend, and the multiplier a finite number.

RULE.

In multiplying, observe to add to the right-hand place of the product, so many units as there are tens in the product of the left-hand place of the repetend; and the product shall contain a repetend, whose places are equal to those in the multiplicand; and if there be more places of figures in the multiplier than one, make all the several products conterminous towards the right hand, as in Case 3.

EXAMPLES.

(19) Multiply 7,$\cancel{6}4\cancel{2}$ by 6. (20) Multiply 64,$\cancel{8}4\cancel{2}$ by 27,9.

Case 7. If the multiplier be a compound repetend.

RULE.

Multiply by each figure of the multiplier as in whole numbers, and add the several products together; then add the result in this manner: set the left-hand figure so many places towards the right-hand as exceed the number of places in the repetend by one, and the rest of the figures in order after it; and thus proceed, till the result last added is beyond the first: lastly, add the several results together, beginning under the right-hand place of the first, and from thence dash as many figures for a repetend as the repetend of the multiplier consists of.

EXAMPLES.

(21) Mul. 46,74 by 4,$\cancel{2}6\cancel{7}$. (22) Mul. 764,742 by ,$\cancel{6}07\cancel{4}$.

Case 8. If both factors have compound repetends.

RULE.

Proceed as in the two last cases; for as the places of the repetend in the product will be uncertain as to their number, they can only be determined, in any manner fit for practice, by continuing and repeating the first product, which will contain a certain repetend, equal to that of the multiplicand.

EXAMPLES.

(23) Mul. 47,$\cancel{6}4\cancel{2}$ by 3,$\cancel{6}7\cancel{4}$. (24) Mul. 76,$\cancel{2}8$ by 8,$\cancel{4}9\cancel{5}$.

L. DIVISION of DECIMALS.

IN division of decimals the work is likewise performed as in whole numbers: the only difficulty is in valuing the quotient, which will be very easy by observing either of the following general

RULES.

1. The first figure in the quotient is always of the same value with that figure of the dividend which answers or stands over the place of units in the divisor.

2. The quotient must always have so many decimal places as the dividend has more than the divisor.

EXAMPLES.

(1) Divide 1735,5 by 6,5.
(2) Divide ,8332 by ,0084.
(3) Divide 186,9 by 7,476.
(4) Divide 6 by ,008.
(5) Div. 246,1476 by 604,25.
(6) Div. 7,268401 by 119.
(7) Divide 6,7258 by 647.
(8) Div. ,0008136 by 678.
(9) Divide 487,67 by 1000.
(10) Div. ,4746 by 10000.

Case 2. There is also a compendious way of contracting the work, reverse to that in page 159, by which much labour is saved, especially when the divisor has many places of decimal parts in it; and is performed by the following

RULE.

By the first rule find what is the value of the first figure in the quotient; then, by knowing the denomination of the first figure, the decimal places may be reduced to any number proposed, by taking as many of the left-hand figures of the dividend as will answer them; and in dividing, omit or point off one figure of the divisor at each operation, that is, for every figure you place in the quotient, point off one in the divisor; having a due regard to the increase which would arise from the figure so omitted, as in Multiplication, Case 2.

EXAMPLES.

(11) Divide 913,08 by 2137,2 and let the quotient only contain three decimal places.

(12) Divide 6109,2674 by 240,649, and let there be only four places of decimals in the quotient.

Case 3. When the dividend is a single or compound repetend.

RULE.

If it be a single repetend, bring down the circulating figure, until the quotient either repeats, or is as exact as required; but if the repetend in the dividend be a compound one, then bring down the circulating figures in the same order they stand in; and when you have got through them all, bring down the first figure in the repetend over again, and so proceed until your quotient either repeat, or be as exact as necessary.

EXAMPLES.

(13) Divide 14$\not{4}$ by 6,84.

(14) Let it be required to divide 1694,6$\not{4}\not{7}$ by 46,840.

Case 4. If the divisor be a single repetend.

RULE.

Place the dividend under itself, but removed one place towards the right-hand; which subtract, and the remainder will be a new dividend; which divide by the divisor in the same manner as if it were a terminate number.

EXAMPLES.

(15) Divide 42,86 by 8.

(16) Divide 6,426 by 6,8

Case 5. If the divisor and dividend consist of terminate numbers joined to the repetend.

RULE.

Place the divisor and dividend under each other, but removed one place towards the right-hand; then subtract the lower lines from the upper: the remainders will be a new divisor and dividend, which proceed with as before directed.

EXAMPLES.

(17) Divide 81869,869$\not{4}$ by 7414,86$\not{4}$.

(18) Divide 9,4694$\not{6}$ by 64,84$\not{7}$.

Case 6. If a compound repetend be found in your divisor, or in both your divisor and dividend.

RULE.

Proceed as in the last case with your divisor and dividend, only remove them each so many places towards the right-hand, as they have places in the repetend of the divisor; but if the divisor be a compound repetend, without any terminate figures, divide by it as a terminate number; first subtracting the dividend from itself, as above directed.

EXAMPLES.

(19) Divide 147,4$\not{2}$683 by 8,$\not{4}$6$\not{7}$.

(20) Divide 4176,426626$\not{8}$ by 37 ,$\not{7}$697$\not{4}$.

LI. REDUCTION of DECIMALS.

Case 1. To reduce a vulgar fraction to a decimal.

RULE.

Add ciphers to the numerator, and divide by the denominator: the quotient will be the decimal fraction required.

EXAMPLES.

(1) Reduce $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$, to decimals.
(2) Reduce $\frac{5}{24}$ to a decimal.
(3) Reduce $\frac{5}{12}$ of $\frac{3}{4}$ to a decimal.
(4) Reduce $\frac{4}{7}$ to a decimal.
(5) Reduce $\frac{2}{3}$ to a decimal.
(6) Reduce $\frac{3}{4}$ of $\frac{1}{2}$ of $\frac{7}{8}$ to a decimal.
(7) Reduce $\frac{1}{4}$ of $\frac{5}{12}$ to a decimal.

Case 2. To reduce coins, weights, measures, &c. into decimals.

RULE I.

Reduce the given money, weights, &c. into the lowest denomination mentioned, for a dividend: then reduce the inte-

ger into the same denomination for a divisor: the result will be the decimal required.

RULE II.

Write the given denominations, or parts, orderly under each other, the inferior or least parts being uppermost; let these be the dividends. Against each part on the left-hand write the number thereof contained in one of its superior; let these be the divisors. Then begin with the lowest denomination, with ciphers added, making a dot between the ciphers and the figure; and divide, writing the quotient of each divisor as decimal parts on the right-hand of the dividend next below it; and let this mixed number be divided by its divisor, and so on, till all be finished; and the last quotient will be the decimal required.

RULE III.

To reduce shillings, pence, and farthings. If the number of shillings be even, take half for the first place of decimals, and let the second and third places be filled up with the farthings contained in the remaining pence and farthings, always remembering to add 1 when they are 25: if 40, add 2. But if the number of shillings be odd, multiply them by 5, and proceed with the pence and farthings as before.

EXAMPLES.

(8) Reduce 17*s.* 6¾*d.* to the decimal of a £.
(9) Reduce 6*s.* 9*d.* to the decimal of a £.
(10) Reduce 9*s.* to the decimal of a guinea.
(11) Reduce 14*s.* 6¾*d.* to the decimal of a moidore.
(12) Reduce 18*s.* 4¾*d.* to the decimal of a £.
(13) Reduce ¾ of a penny to the decimal of a £.
(14) Reduce 11 dwts. to the decimal of a lb. Troy.
(15) Reduce 10 drams to the decimal of a lb. avoirdupois.
(16) Reduce 3 qrs. 14 lb. to the decimal of a cwt.
(17) Reduce 6 inches to the decimal of a yard.
(18) Reduce 6 furlongs to the decimal of a league.

(19) Reduce 18 galls. 2 qrts. of wine to the decimal of a hhd.
(20) Reduce 3 qrts. 1 pt. of ale to the decimal of a barrel.
(21) Reduce 8 perches to the decimal of an acre.
(22) Reduce 4 bushels 2 pecks to the decimal of a chaldron.
(23) Reduce 12 minutes to the decimal of an hour.
(24) Reduce 12 days to the decimal of a Julian year.

By this rule the following decimal tables are made.

Decimal TABLES *of* COIN, WEIGHT, *and* MEASURE.

TABLE I.

COIN.

1 £. Ster. the Integ.

Sh.	*dec.*	*Sh.*	*dec.*
19	,95	9	,45
18	,9	8	,4
17	,85	7	,35
16	,8	6	,3
15	,75	5	,25
14	,7	4	,2
13	,65	3	,15
12	,6	2	,1
11	,55	1	,05
10	,5		

Pence.	*Decimals.*
11	,045833
10	,041666
9	,0375
8	,033333
7	,029166
6	,025
5	,020833
4	,016666
3	,125
2	,008333
1	,00416

Farth.	*Decimals.*
3	,003125
2	,002083
1	,001042

TABLE II.

TROY WEIGHT.

1 lb. the Integer.

Ounces	*Decimals.*
11	,916666
10	,833333
9	,75
8	,666666
7	,583333
6	,5
5	,416666
4	,333333
3	,25
2	,166666
1	,083333

Note. This Table of Oz. will also serve for Inches, Months, or Doz.

Penny-weight.	*Decimals.*
10	,041666
9	,0375
8	,033333
7	,029166
6	,025
5	,020833
4	,016666
3	,125
2	,008333
1	,004166

Grains.	*Decimals.*
20	,003472
10	,001735
9	,001562
8	,001389
7	,001213
6	,001042
5	,000868
4	,000694
3	,000521
2	,000347
1	,000173
½	,000086

TABLE III.

AVOIRDUPOIS.

112 lb. the Integer.

Quart.	*Decimals.*
1	,25
2	,5
3	,75

Lb.	*Decimals.*
20	,178571
10	,089286
9	,080357
8	,071428
7	,0625
6	,053571
5	,044643
4	,035714
3	,026786
2	,017857
1	,008928

Ounces.	*Decimals.*
10	,00558
9	,005022
8	,004464
7	,003906
6	,003348
5	,00279
4	,002232
3	,001673
2	,001116
1	,000558

Drams.	*Decimals.*
10	,000348
9	,000313
8	,000279
7	,000244
6	,000209
5	,000174
4	,000139

Decimal Tables of Coin, Weight, and Measure.

3	,000104
2	,000069
1	,000034
½	,000017

TABLE IV.
Avoir. Weight.
1 lb. the Integer.

Ounces.	Decimals.
8	,5
7	,4375
6	,375
5	,3125
4	,25
3	,1875
2	,125
1	,0625

Drams.	Decimals.
8	,03125
7	,027343
6	,023427
5	,019531
4	,015625
3	,011718
2	,007812
1	.003906

TABLE V.
Liq. Measure.
1 Tun the Integer.

Gallons	Decimals.
100	,396825
90	,357141
80	,317462
70	,27
60	,238095
50	,198412
40	,15873
30	,119047
20	,079365
10	,039682
9	,035714
8	,031746
7	,027
6	,023809
5	,019841
4	,015873
3	,011904
2	,007936
1	,003968

Pints.	Decimals.
4	,001984
3	,001488
2	,000992
1	,000496

Hogshead the Integer.

Gallons.	Decimals.
30	,47619
20	,31746
10	,15873
9	,142857
8	,126984
7	,111111
6	,095238
5	,079365
4	,063492
3	,047619
2	,031746
1	,015873

Pints.	Decimals.
3	,005952
2	,003968
1	.001984

TABLE VI.
Measure.

Liquid.	Dry.
1 Gallon	1 Quarter

Integers.

Pints.	Decim.	Bush.
4	,5	4
3	,375	3
2	,25	2
1	,125	1

Q. P.	Decim	Pk.
3	,09375	3
2	,0625	2
1	,03125	1

Decimals.	Q. Pk.
,023432	3
,025615	2
,026812	1

Decimals.	Pints.
,005859	3
,003900	2
,001053	1

TABLE VII.
Long Measure.
1 Mile the Integer.

Yards	Decimals.
1000	,568182
900	,511364
800	,454545
700	,397727
600	,340909
500	,284091
400	,227272
300	,170454
200	,113636
100	,056818
90	,051136
80	,045454
70	,039773
60	,054091
50	,028409
40	,022727
30	,017045
20	,011364

Decimal Tables of Coin, Weight, and Measure.

10	,005682
9	,005114
8	,004545
7	,003977
6	,003409
5	,002841
4	,002273
3	,001704
2	,001139
1	,000568

Feet.	*Decimals.*
2	,0003787
1	,0001894

Inches.	*Decimals.*
6	,0000947
3	,0000474
2	,0000315
1	,0000158

TABLE VIII.

Time.

1 Year the Integer.

Days.	*Decimals.*
300	,821918
200	,547945
100	,273963
90	,246575
80	,219178
70	,191781
60	,164383
50	,136986
40	,109589
30	,082192
20	,054794
10	,027397
9	,024657
8	,021918
7	,019178
6	,016438
5	,013609
4	,010959
3	,008219
2	,005479
1	,002739

1 Day the Integer.

Hours.	*Decimals.*
20	,833333
10	,416666
9	,375
8	,333333
7	,291666
6	,25
5	,208333
4	,166666
3	,125
2	,083333
1	,041666

Min.	*Decimals.*
50	,034722
40	,027777
30	,020833
20	,013888
10	,006944
9	,00625
8	,005555
7	,004861
6	,004166
5	,003472
4	,002777
3	,002083
2	,001388
1	,000694

TABLE IX.

Cloth Measure.

1 Yard the Integer.

Qrs.	*Decimals.*
3	,75
2	,5
1	,25

Nails.	*Decimals.*
3	,1875
2	,125
1	,0625

TABLE X.

Lead Weight.

1 Foth. the Integer.

Hund.	*Decimals.*
10	,51282
9	,461538
8	,410256
7	,358974
6	,307692
5	,25641
4	,205128
3	,153846
2	,102564
1	,001282

Qrs.	*Decimals.*
2	,025641
1	,01282

Lbs.	*Decimals.*
14	,0064102
13	,0059523
12	,0054945
11	,0050366
10	,004578
9	,0041208
8	,003663
7	,0032051
6	,0027472
5	,0022893
4	,0018315
3	,0013736
2	,0009157
1	,0004578

Case 3. To find the Value of any Decimal Fraction, in Money, Weight, Measure, &c.

RULE.

Multiply the decimal by the number of parts of the next inferior denomination, cutting off so many places for decimals to the right hand as your given decimal consists of; and those on the left will be integers; then multiply the remaining decimals by the next inferior denomination, and cut off for decimals as before; thus proceed till you have brought it into the least parts of the integer.

EXAMPLES.

(25) What is the value of ,878125 of a pound sterling?
(26) What is the value of ,3375 of a £.?
(27) What is the value of ,45 of a guinea?
(28) What is the value of ,72708 of a moidore?
(29) What is the value of ,00243 of a lb. Troy?
(30) What is the value of ,3375 of a ton?
(31) What is the value of ,0396 of a lb. avoirdupois?
(32) What is the value of ,875 of a cwt.?
(33) What is the value of ,16669 of a yard?
(34) What is the value of ,259 of a league?
(35) What is the value of ,29365 of a hhd. of wine?
(36) What is the value of ,875 of a barrel of ale?
(37) What is the value of ,05 of an acre?
(38) What is the value of ,125 of a chaldron of coals?
(39) What is the value of ,4765 of a day?

LII. EXTRACTION of the SQUARE ROOT.

EXTRACTING the Square Root is finding out such a Number, as, being multiplied into itself, the Product will be equal to the given Number.

As the Square Root of 81 is 9, consequently $9 \times 9 = 81$ the given Number, as in the following Table.

Roots	1	2	3	4	5	6	7	8	9
Squares	1	4	9	16	25	36	49	64	81

Case 1. To extract the Square Root of any Number, observe the following

RULE.

1. Point the given number or resolvend into periods of two figures, beginning at the unit's place.

2. Find by the Table the greatest square number that is contained in the first period towards the left hand, placing the square number under the first period, and the root thereof in the quotient, as in Division; subtract that square out of the said period, and to the remainder bring down the next period for a dividend.

3. Double the quotient or root, and place it for a divisor, seek how often the divisor is contained in the dividend (reserving always the unit's place), and put the answer in the quotient, and also on the right hand of the divisor. Then multiply the divisor by the last figure put in the quotient, as in common Division, subtract the product from the dividend, and to the remainder bring down the next period, which proceed with as before.

Note 1. But if it happen that the given resolvend is not a perfect square, cube, &c. then something will remain after extraction has been made throughout all the points. When this is the case, you must annex ciphers, according as the proposed power requires, viz. by pairs or twos in the square; threes in the cube, &c.; and the operation continued as before.

2. If the given resolvend consist of a whole number and decimals together, make the number of decimals even by adding ciphers to them.

EXAMPLES.

(1) It is required to extract the square root of 74770669.
(2) What is the square root of 60516?
(3) What is the square root of 7658?
(4) What is the square root of 39342864?
(5) What is the square root of 8209667940,829?
(6) What is the square root of ,000729?
(7) What is the square root of 2?
(8) What is the square root of 2,2710957?
(9) What is the square root of 36,00000625?

LIII. To extract the SQUARE ROOT of VULGAR FRACTIONS.

RULE.

Reduce the fraction or fractional parts to its lowest terms, and if it be a mixed number, to an improper fraction; then extract the square root of the numerator for a new numerator, and the square root of the denominator for a new denominator.

EXAMPLES.

(1) What is the square root of $\frac{25}{81}$?
(2) What is the square root of $\frac{225}{324}$?
(3) What is the square root of $10\frac{39}{49}$?
(4) What is the square root of $27\frac{9}{16}$?

SURDS.

Case 3. To extract the square roots of vulgar fractions, when they are surds, *i. e.* a number where a root can never be exactly found.

RULE.

Reduce the fraction or fractional part to its lowest terms, then reduce it to a decimal, and annex that decimal to the whole number (if any), and extract the square root therefrom.

EXAMPLES.

(5) What is the square root of $8\frac{3}{7}$?
(6) What is the square root of $\frac{197}{576}$?
(7) What is the square root of $76\frac{14}{17}$?

LIV. The USE of the SQUARE ROOT.

Case 1. To find a mean proportional between any two given numbers.

RULE.

Multiply the two given numbers together, and extract the square root of the product, which root will be the mean proportional sought.

EXAMPLES.

(1) What is the mean proportional between 4 and 9?
(2) What is the mean proportional between 16 and 36?

Case 2. To find the side of a square equal in area to any given superficies.

RULE.

Extract the square root of the given superficies, which root will be the side of the square sought.

EXAMPLES.

(3) If the area of a given circle be 4276,5, I demand the side of a square, whose superficial content shall be equal thereto.
(4) Suppose I have an elliptical or irregular fish-pond, containing in surface 9 acres, 2 roods, 15 perches, and would have a square one of the same content; I desire you will tell how many yards each side must be.
(5) If the content of a given circle be 160, what is the side of a square equal thereto?

Case 3. Having the area of a circle, to find the diameter.

RULE.

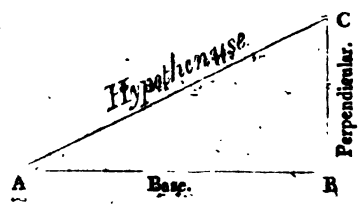
As 355 : 452 :: or, as 1 : 1,273239 :: so is the area : to the square of the diameter; or multiply the square root of the area by 1,12837, and the product will be the answer. (See Problem VI. in Mensuration.)

EXAMPLES.

(6) Required the diameter of a circle that will comprehend within its circumference the quantity of an acre of land.

(7) In the midst of a meadow well stored with grass,
I took just two acres to tether my horse;
How long must the cord be, that feeding all round,
He mayn't graze less or more than these two acres of ground?

Case 4. Any two sides of a right-angled triangle, A B C, being given, to find the remaining side.



1. The base and perpendicular being given, to find the hypothenuse.

RULE.

Square each side, add the squares together, and the square root of this sum gives the hypothenuse required.

2. If the hypothenuse and one side be given, to find the other side.

RULE.

From the square of the hypothenuse subtract the square of the given side; the square root of the remainder gives the side required.

EXAMPLES.

(8) At Matlock, near the Peak in Derbyshire, where are many surprising curiosities of nature, is a rock by the side of the river Derwent, rising perpendicularly to

a wonderful height, which being inaccessible, I endeavoured to measure, and found, by a mathematical method, that the distance between the place of observation and the foot of the rock was $55\frac{1}{4}$ yards, and from the top of the rock to the said place was $140\frac{1}{2}$ yards, nearly. Required the height of this stupendous rock.

(9) A ladder 40 feet long may be so planted that it shall reach a window 33 feet from the ground on one side the street; and, without moving it at the foot, will do the same by a window 21 feet high on the other side. The breadth of the street is required.

(10) A line, 27 yards long, will exactly reach from the top of a fort to the opposite bank of a river, known to be 23 yards broad. The height of the wall is required.

(11) Suppose a light-house built on the top of a rock; the distance between the place of observation and that part of the rock level with the eye, and directly under the building, is given 310 fathoms; the distance from the top of the rock to the place of observation is 423 fathoms; and from the top of the building 425. The height of the edifice is required.

(12) Two ships set sail from the same port; one of them sails due east 50 leagues, the other due north 84. How far are they asunder?

QUESTIONS *for Exercise at leisure Hours.*

(13) The height of an elm, growing in the middle of a circular island 30 feet in diameter, plumbs 53 feet; and a line, stretched from the top of the tree straight to the hither edge of the water, 112 feet. What then is the breadth of the moat, supposing the land on the other side the water to be level?

(14) Required the length of a shore, that being to strut 11 feet from the upright of a building, will support a jamb 23 feet ten inches from the ground.

(15) There are two columns in the ruins of Persepolis, left standing upright; one is 64 feet above the plane, the other 50. Between these, in a right line, stands an ancient statue, the head whereof is 97 feet from the summit of the higher, and 86 feet from the top

of the lower column; the base whereof measures just 76 feet to the centre of the figure's base. By these notices, the distance of the top of the columns may be, by numbers, easily found.

(16) A castle wall there was, whose height was found
To be a hundred feet from th' top to th' ground;
Against the wall a ladder stood upright,
Of the same length the castle was in height.
A waggish youth did the ladder slide
(The bottom of it) ten feet from the side:
Now I would know, how far the top did fall,
By pulling out the ladder from the wall.

(17) As I was walking out one day,
Which happen'd on the first of May,
As luck would have it, I did spy
A May-pole raised up on high,
The which at first me much surpris'd,
Not being before-hand advertis'd
Of such a strange uncommon sight;
I said I would not stir that night,
Nor rest content, until I'd found
Its height exact from off the ground.
But when these words I just had spoke,
A blast of wind the May-pole broke,
Whose broken piece I found to be
Exact in length yards sixty-three,
Which by its fall broke up a hole,
Twice fifteen yards from off the pole;
But this being all that I can do,
The May-pole now being broke in two
Unequal parts, to aid a friend,
Ye youths, pray then an answer send.

Case 5. Any number of men being given, to form them into square battle, or to find the number of ranks and files.

RULE.

Extract the square root of the number of men given, will give the number of men either in rank or file.

EXAMPLE.

(18) A general disposing his army into a square battle, finds he has 23716 men; required the number in rank and file.

LV. The EXTRACTION of the CUBE ROOT.

To extract the cube root is to find out a number, which being multiplied into itself, and then again into the product, produces the given number.

As the cube root of 729 is 9, consequently $9 \times 9 \times 9 = 729$ the given number, and so of others, as in the following Table.

Roots	1	2	3	4	5	6	7	8	9
Cube	1	8	27	64	125	216	343	512	729

RULE.

1. Make a point over every third figure given, beginning at the unit's place; seek the greatest cube to the first point on the left hand (by the Table), whose root place in the quotient; then subtract its cube from the period, and to the remainder (if any) bring down the three figures, or your next period, and call it your dividend.

2. Find a divisor, by calling your quotient figure, with a cipher joined to it, *r*; then three times the square of *r* will be your divisor; seek how often it is contained in the dividend, and put the answer in the quotient as in Division, only with this difference; call the said quotient figure last put up *e*, and multiply your divisor by it, and place the produce underneath the dividend; then multiply the square of *e* by three times *r*, and place it also under the dividend; lastly, cube the figure you call *e*, and place it under the dividend: then add the three products together, gives the subtrahend, which subtract from your last dividend, and to the remainder bring down the next period, and proceed as before.

EXAMPLES.

(1) What is the cube root of 21624576?
(2) Extract the cube root of 92398647.
(3) What is the cube root of 2716243264?
(4) What is the cube root of 91?
(5) What is the cube root of 67527834239?
(6) Extract the cube root of 4764,75.
(7) The solidity of a cube is 36155,027576 inches, what is the side of that cube?
(8) What is the side of that cube which contains 67667,921875 solid inches?
(9) What is the cube root of 219563329?
(10) What is the cube root of 3105926,917?
(11) What is the cube root of ,000421875?
(12) What is the side of a cube whose solidity is 28022810,390625?

The biquadrate of any number is found by extracting the square root of the given number first, and then the square root of that root.

(13) Let it be required to extract the biquadrate of 4857532416.

The root of the square cubed, or sixth power of any number, is found by extracting the square root of the given number, then extract the cube root of that square root, which will give the sixth power required.

(14) Let it be required to extract the square cubed root of 49656.

The root of the biquadrate squared, or eighth power, is found by first extracting the square root of the given number, which will reduce it to a biquadrate, which proceed with as before directed.

(15) Let it be required to extract or find the root of the eighth power of 43046721.

The root of the cube cubed, or ninth power of any number, is found by extracting the cube root of the given number, and the result will be a cubic resolvend: of this extract the cube root also, which will be the root of the ninth power.

(16) Let it be required to extract or find the root of the ninth power of 387420489.

The extraction of the first and second surolids, i. e. the fifth and seventh powers, will prove too difficult a task in common numbers; I would therefore advise the tutor to teach his pupils only the square and cube roots here, as they are the most useful, and, in short, as high as is required in all common things. Besides, the reason of the thing cannot be shown till the pupil come to the algebraic part, where it will be easily performed, and appear more evident.

LVI. To extract the CUBE ROOT of a VULGAR FRACTION.

RULE.

The same rules which are given in page 172 must be observed here, only extracting the cube root instead of the square; that is, reduce the fractions to their lowest terms; if it be a mixed number, to an improper fraction; and if a surd, to a decimal.

EXAMPLES.

(1) What is the cube root of $\frac{384}{1296}$?
(2) What is the cube root of $\frac{352}{1188}$?
(3) What is the cube root of $5\frac{104}{125}$?
(4) What is the cube root of $405\frac{28}{512}$?

SURDS.

(5) What is the cube root of $5\frac{3}{5}$?
(6) What is the cube root of $7\frac{6}{7}$?

LVII. The USE of the CUBE ROOT.

Case 1. To find the side of a cube that shall be equal in solidity to any given solid, as a globe, cylinder, prism, cone, &c.

RULE.

Extract the cube root of the solid content of the given body, which root will be the side of the cube required.

EXAMPLES.

(1) There is a stone of a cubic form, which contains 219 solid feet. What is the superficial content of one of its sides?

Case 2. Having the dimensions of any solid body, to find the dimensions of another similar solid, that shall be any number of times greater or less than the solid given.

RULE.

Multiply the cube of each side by the difference between the solid given and that required, if greater (or divide by the difference, if less) than the solid given; then extract the cube root of each product or quotient, which will give the dimensions of the solid required.

EXAMPLES.

(2) Suppose the length of a ship's keel to be 125 feet, the breadth of the midship beam 25 feet, and the depth of the hold 15 feet: I demand the dimensions of another ship of the same form, that shall carry three times the burden.

(3) Again, I demand the dimensions of another ship of the same form, that shall only be half the burden of that whose dimensions are given as above.

Case 3. Having the dimensions and capacity of a solid, to find the dimensions of a similar solid of a different capacity.

RULE.

Like solids are in triplicate proportion to their homologous sides; therefore it will be, As the cube of a dimension : is to its given weight :: so is the cube of any like dimension : to the weight sought.

EXAMPLES.

(4) If a ship of 300 tons burden be 75 feet long in the keel, I demand the burden of another ship whose keel is 100 feet long.

(5) Suppose a ball of 4 inches diameter weigh 18 lb. I demand the diameter of another that weighs 114 lb.

(6) If a brass cannon, whose diameter is 11,5 inches, weigh 1000 lb. what will another piece of ordnance of the same metal and shape, weigh, whose diameter is 20,83 inches?

Case 4. To find two mean proportionals between two given Numbers.

RULE.

Divide the greater extreme by the less; and the cube root of the quotient multiplied by the less extreme, gives the lesser mean: multiply the said cube root by the lesser mean, and the product will be the greater mean proportional.

EXAMPLES.

(7) What are the two mean proportionals between 7 and 189?

(8) Find two mean proportionals between 4 and 256.

LVIII. The SINGLE RULE of THREE in DECIMALS.

RULE.

REDUCE the fractional parts into decimals of the highest name mentioned; then state the questions, and proceed as in whole Numbers, Sections XII. and XIII.

EXAMPLES.

(1) Suppose I give 6*s.* 3*d.* for 4¾ yards of cloth; what will 48¾ yds. of the same come to at that rate?

(2) If 2¾ lb. of tea cost 1*l.* 5*s.* what will 14¾ lb. come to at the same rate?

(3) If 1 lb. of sugar cost 11¾*d.* what will 4 hhds. each weighing net 4 cwt. 3 qrs. 14 lb. cost at the same rate?

(4) A grocer buys 4 chests of tea, each weighing net 2 cwt. 3 qrs. 14 lb. for 906*l.* 10*s.* At what rate did he give per lb.?

(5) How far will a person be able to travel in 9 days 8 hours, at the rate of 12 miles every 4 hours, allowing 12 hours to the day?

(6) An oilman bought 4 tuns 20$1\frac{1}{2}$ gallons of Florence oil for 240*l.* 16*s.* 6*d.*; but by misfortune it chanced to leak out 24$\frac{1}{2}$ gallons. I desire to know at what he must sell the remainder per gallon to be no loser.

(7) Goliah is said to have been 6 cubits and a half, or a span, high; this answers to 10 feet 4 inches and $\frac{74}{125}$. Pray what was the length of the cubit in British measure?

QUESTIONS *for Exercise at leisure Hours.*

(8) In a series of proportional numbers, the first is 5, the third 8; the product of the second and third is 78,4. What is the difference of the second and fourth?

(9) If the cubic inch of olive oil be ,52835 decimal parts of an ounce avoirdupois, what quantity of oil, weighing $7\frac{1}{2}$ lb. per gallon, will be contained in a cask allowed to hold $13\frac{1}{3}$ gallons of water, each 282 solid inches?

(10) The cubic inch of marble is 1,5688 oz. avoirdupois. What difference is there, in point of weight, between a figure containing a solid foot and a half of stone, and another of equal dimensions in brass, 4,63 oz. whereof make a cubic inch.

(11) There are two numbers, the lesser 75, to which the greater is in proportion as 8 to 5. What is their sum, and the product of their sum and difference, the difference and product of their squares, and the sum of the square of their two quotas, the greater divided by the less, and again the less by the greater?

(12) There are two numbers more, the greater, 224, bearing proportion to the other as 8 to 7. What is the square of their sum, difference, and either quota? What is the result of the square of the sum of the difference, added to the product of their sum and difference?

(13) If, during the tide of ebb, a wherry should set out from London westward, and at the same instant another should put off at Chertsey for London, taking the distance by water at 34 miles: the stream forwards one and retards the other, say, $2\frac{1}{2}$ miles an hour. The boats are equally laden, the rowers equally good, and in the ordinary way of working, in still water, would proceed at the rate of 5 miles

an hour. The question is, where in the river the two boats would meet.

(14) A bullet of cast iron, 4 inches diameter, weighs experimentally 9 lb. What is the difference of the weight of one that is $13\frac{1}{2}$ inches in diameter, and another that is no more than $7\frac{1}{4}$ inches?

(15) A gay young fellow had 18,200*l.* left him by an old uncle, to whose memory he expended three per cent. of his whole fortune, in a sumptuous funeral and monument: 9 per cent. of the remainder he made a present of to his cousins, forgotten for his sake by the old man; with $\frac{3}{7}$ of what was left he bought a fine seat: with $\frac{1}{5}$ of the residue a stud of horses: he squandered away 550*l.* upon one mistress; and after he had lived at the rate of 2000*l.* a-year for 19 months together, he had both ruined his health and impaired his fortune. Pray, at his death, what was there left for his sister, who was his heir at law?

The EFFECTS of LIGHT, HEAT, and ATTRACTION.

THE effects or degrees of light, heat, and attraction, are reciprocally proportional to the square of their distances from the centre whence they are propagated.

(16) Suppose that in a room, where two men, A. and B. are sitting, there is a fire, from which A. is three feet, and B. is six feet distant; it is required to find how much hotter it is at A.'s seat than at B.'s.

(17) Supposing the earth to be 81000000 miles distant from the sun, I would know at what distance from him another body must be placed, so as to receive light and heat double to that of the earth.

(18) The distance between the earth and sun is accounted 81000000 miles; the distance between Jupiter and the sun is 424000000 miles. The degree of light and heat received by Jupiter, compared with that of the earth, is required.

(19) Mercury, the nearest of the planets to the source of heat, light, and life, in our system, the sun, is about 32 millions of miles from him; Saturn, the remotest of the planets, is usually distant about 777 millions of miles. What comparison or proportion is there between the solar influences on these two bodies?

(20) Suppose, with Dr. Keil, the distance of the sun to be from us 115 of his diameters; how much hotter is it then at the surface of the sun than under our equator?

(21) A ball, descending by the force of gravity from the top of a tower, was observed to fall half the way in the last second of time. Required the tower's height, and the whole time of descent.

The less porous a body is, the greater is its density.

(22) The compactness or density of the moon is to that of the earth, as $132\frac{1}{2}$ is to 100. What proportion then is there between the quantity of matter in the earth and that in the moon, since the earth's diameter is 7970 miles, and that of the moon 2170?

(23) There is a vast country in Ethiopia Superior, to whose inhabitants the moon always appears to be most enlightened when she is least enlightened, and to be least when most, according to Gordon's Geographical Grammar. Admitting the mean distance of the earth and moon's centres 240,000 miles, in what proportion is this illumination?

Velocities acquired by heavy Bodies falling.

The velocity acquired by heavy bodies falling near the surface of the earth is $16\frac{1}{2}$ feet in the first second: and as $16\frac{1}{2}$ feet are to the square of one second, or 1, so is the given distance to the square of the seconds required: or, on the contrary, to determine what space a heavy body has passed in any time given, is,

By multiplying $16\frac{1}{2}$, the descent of a heavy body in one second of time, by as many of the odd numbers, beginning from unity, as there are seconds in the given time, viz. by 1 for the first, 3 for the second, 5 for the third, 7 for the fourth, &c.: the sum total will give the space it has passed.

(24) Suppose a stone let go into an abyss should be stopped at the end of the eleventh second after its delivery, what space would it have gone through?

(25) What is the difference between the depth of two wells, into each of which should a stone be dropped at the same instant, one will meet with the bottom at 6 seconds, the other at 10?

(26) If a stone be $19\frac{1}{2}$ seconds in descending from the top

of a precipice to the bottom, what is the height of the same?

(27) In what time would a musket ball, dropped from the top of Salisbury steeple, said to be 400 feet high, be at the bottom?

(28) If a hole could be bored through the centre of the earth, in what time after the delivery of a heavy body on its surface would it arrive at its centre?

LIX. The DOUBLE RULE of THREE in DECIMALS.

REDUCE the fractional parts to decimals, and then proceed as in whole numbers.

EXAMPLES.

(1) If 1*l.* 2*s.* worth of wine will suffice a club of 12 persons when the wine is sold at the rate of 25*l.* 4*s.* per hhd. how many persons will 1*l.* 12*s.* worth serve, when the wine is sold after the rate of 18 guineas per hhd.

(2) If 6 lb. of pepper be worth 13 lb. of ginger, and 19 lb. of this be worth 4¼ lb. of cloves, and 10 lb. be equivalent to 63 lb. of sugar at 5*d.* per lb. what is the value of 1 cwt. of pepper?

(3) What money, at 3½ per cent. will clear 38*l.* 10*s.* in a year and a quarter's time?

QUESTIONS *for Exercise at leisure Hours.*

(4) A. lent his good friend B. fourscore and eleven guineas, from the 11th of Dec. to the 12th of May following; B. on another occasion, let A. have 100 marks, from September 3 to Christmas following. Quere, how long ought the person obliged to let his friend use 40*l.* fully to retaliate the favour?

(5) A. B. and C. will trench a field in 12 days; B. C. and D. in 14; C. D. and A. will do it in 15, and D. A. and B. in 18. In what time will it be done by all of them together, and by each of them singly?

(6) A young hare starts 5 rods before a greyhound, and is not perceived by him till she has been up 34 seconds; she scuds away at the rate of 12 miles an hour; and the dog, on view, makes after her at the rate of 20.

How long will the course hold, and what ground will be run, beginning with the outsetting of the dog?

VIBRATION of PENDULUMS.

IT has been found by experiment, that a pendulum 39,2 inches long, in our latitude, vibrates 60 times in one minute; and that the lengths of the pendulums are to one another reciprocally as the square of the number of their vibrations made in the same space of time.

(7) What difference is there between the length of a pendulum that vibrates half a second, or 120 times in a minute, and another that swings double seconds, or 30 times in a minute?

(8) What difference will there be in the number of vibrations made by a pendulum of 6 inches long, and another of 12 inches long, in an hour's time?

(9) What difference is there in the length of two pendulums, the one swinging 30 times, the other 100 times in an hour?

(10) Give the length of a pendulum that will swing once in a third, ditto in a second, ditto in a minute, ditto in an hour, ditto in a day.

(11) Observed, that while a stone was descending to measure the depth of a well, a string and plummet, that from the point of suspension, or the place where it was held, to the centre of oscillation, or that part of the bob, which, being divided by the circular line, struck from the above centre, would divide it into two parts of equal weight, measured just 18 inches, had made 8 vibrations. Pray what was the depth, allowing 1150 feet per second for the return of sound to the ear?

LX. FELLOWSHIP.

How to perform fellowship, either single or double, without that tedious and laborious task of making so many different statings as there are persons concerned.

RULE.

1. DIVIDE the whole gain or loss by the whole stock.

2. Multiply the quotient by each person's particular stock; and the several products will be the respective gain or loss of each.

Note.—This rule is best adapted for decimals.

EXAMPLES.

(1) Three persons making a joint stock: A. puts in 750*l.* B. 450*l.* C. 300*l.* with which they trade a certain time; and, when they balance accounts, find that they have gained 300*l.* What is the share of each?

(2) Three merchants, A, B, and C. traded together; A. put in 120*l.* for 8 months; B. 250*l.* for 4 months; and C. 100*l.* for 5 months; they gained 184*l.* 10*s.* What is each man's share of the gain?

(3) Once as I walk'd upon the banks of the Wye,
To see the purling streams glide gently by,
And hear the pretty birds to chirp and sing,
Making the groves with melody to ring;
I, in the meads, three beauteous nymphs did spy,
That for their pleasure came as well as I;
And unto me their steps they did direct,
Saluting me with most benign respect,
Saying, "Well met! we've business to impart,
Which we cannot decide without your art.
Our grannam's dead, and left a legacy,
Which is to be divided amongst three:
In pounds it is two hundred twenty-nine,
Also a good mark, being sterling coin."
Then spake the eldest of the lovely three,
"I'll tell you how it must divided be;
Likewise our names I unto you will tell;
My name is Moll, the others' Anne and Nell.
As oft as I five and five-ninths do take,
Anne takes four and three-sevenths her part to make;
As oft as Anne four and one-ninth does tell,
Three and two-thirds must be took up by Nell."

For more examples, see Sections XXV. and XXVI.

Of Simple Interest, Annuities, or Pensions, &c.

LXI. SIMPLE INTEREST.

Here are five letters to be observed, viz.

P=any principal or sum put to interest.
I=the interest.
T=the time of the principal's continuance at interest.
A=the amount, or principal and interest.
R=the ratio, or rate per cent. per annum.

Note.—The ratio is the simple interest of 1*l.* for one year, at any given rate; and is thus found,

Viz. 100 : 5 :: 1 : ,05 the ratio at 5 per cent. per ann.
Or, 100 : 6 :: 1 : ,06 the ratio at 6 per cent. per ann. &c.

And in this manner the ratios in the following table are found.

TABLE.

3 =,03	$4\frac{1}{2}$=,045
$3\frac{1}{2}$=,035	5 =,05
4 =,04	$5\frac{1}{2}$=,055

Case 1. When the principal, time, and rate per cent. are given, to find the interest.

RULE.

Multiply the principal, rate, and time, continually into one another, the product is the interest sought.

Or, if p=the principal, t=the time, r=the rate, and I=the interest, then the theorem will be as follows:

THEOREM 1. $p\ t\ r$=I.

EXAMPLES.

(1) What is the interest of 260*l.* 17*s.* 6*d.* for $5\frac{1}{2}$ years at $4\frac{1}{2}$ per cent. per annum?

(2) What is the interest of 500*l.* from May the 12th, 1784, to November the 24th, 1789, at $3\frac{3}{4}$ per cent. per ann.?

Case 2. When the interest required is for days only.

RULE.

Multiply the interest of 1*l.* for one day, at the given rate, by the principal and number of days; it will give the answer.

The interest of 1*l.* for one day is thus found:

	d.	*l.*		*d.*	*l.*	
Viz. As	365 :	,05	::	1 :	,0001369863,	&c.
Or	365 :	,035	::	1 :	,00009589041,	&c.

TABLE.

per Cent.	*Decimals.*
3	=,00008219178
$3\frac{1}{2}$	=,00009589041
4	=,00010958904
$4\frac{1}{2}$	=,00012328767
5	=,0001369863

EXAMPLES.

(3) What is the interest of 370*l.* 10*s.* for 220 days, at $4\frac{1}{2}$ per cent. per annum?

(4) What is the interest of 600*l.* from the 1st of July, 1789, to the 24th of February following, at 6 per cent.?

Case 3. When the principal, time, and rate per cent. are given, to find the amount.

RULE.

Find the interest by Theorem 1, which, added to the principal, will give the amount.

Thus, THEOREM 2. $p\ t\ r. + p = A$.

EXAMPLES.

(5) What will 234*l.* 10*s.* amount to in 7 years, at $3\frac{1}{2}$ per cent. per annum?

(6) What will 612*l.* 5*s.* amount to in $5\frac{1}{2}$ years, at $4\frac{1}{2}$ per cent. per annum?

(7) What will 500*l.* amount to in 6 years 120 days, at $4\frac{3}{4}$ per cent. per annum?

Case 4. When the rate, time, and interest are given, to find the principal.

RULE.

Divide the interest by the product of rate and time, the quote is the principal.

Thus, THEOREM 3. $\frac{I}{tr} = p$.

EXAMPLES.

(8) I demand what principal, being put to interest for 3 years, will gain 69*l.* 13*s.* 6*d.* at 5 per cent. per ann.

(9) I demand what principal, being put to interest for $5\frac{1}{2}$ years, will gain 64*l.* 7*s.* at $4\frac{1}{2}$ per cent. per ann.

(10) I demand what principal, being put to interest for 4 years, at 4 per cent. will gain 67*l.* 15*s.* $9\frac{3}{4}$*d.*

Case 5. When the amount, rate, and time are given to find the principal.

RULE.

Add 1 to the product of the rate and time, and by that sum divide the amount: the quote is the principal.

Thus, THEOREM 4. $\frac{a}{tr+1} = p$.

EXAMPLES.

(11) What principal, being put to interest, will amount to 354*l.* 4*s.* $0\frac{1}{2}$*d.* in 7 years, at $3\frac{1}{2}$ per cent. per annum?

(12) What principal, being put to interest, will amount to 500*l.* 9*s.* $3\frac{3}{4}$*d.* in 6 years 5 months, at 5 per cent. per annum?

(13) What principal, being put to interest for 7 years 220 days, at $4\frac{3}{4}$ per cent. per annum, will amount to 100*l.*?

Case 6. When the principal, interest, and rate are given, to find the time.

RULE.

Divide the interest by the product of the principal and rate: the quote is the time.

Thus, THEOREM 5. $\frac{I}{pr}=t$.

EXAMPLES.

(14) In what time will 464*l.* 10*s.* gain 69*l.* 13*s.* 6*d.* at 5 per cent. per annum?

(15) In what time will 260*l.* gain 64*l.* 7*s.* at $4\frac{1}{2}$ per cent. per annum?

(16) In what time will 500*l.* gain 130*l.* 9*s.* 7*d.* at $6\frac{1}{2}$ per cent. per annum?

Case 7. When the principal, interest, and rate are given, to find the time.

RULE.

Divide the amount, less the principal, by the product of the principal and rate: the quote is the time.

Thus, THEOREM 6. $\frac{a-p}{pr}=t$.

EXAMPLES.

(17) In what time will 284*l.* 10*s.* amount to 354*l.* 4*s.* $0\frac{1}{2}$*d.* at $3\frac{1}{4}$ per cent. per annum?

(18) In what time will 672*l.* 5*s.* amount to 847*l.* 17*s.* 6*d.* at $4\frac{3}{4}$ per cent. per annum?

(19) In what time will 378*l.* 18*s.* amount to 500*l.* 9*s.* $3\frac{1}{2}$*d.* at 5 per cent. per annum?

Case 8. When the principal, interest, and time are given, to find the rate per cent.

RULE.

Divide the interest by the product of the principal and time: the quote is the rate.

Thus, THEOREM 7. $\frac{I}{pt}=r$.

EXAMPLES.

(20) At what rate per cent. will 464*l.* 10*s.* gain 69*l.* 13*s.* 6*d.* in 3 years?

(21) At what rate per cent. will 260*l.* gain 64*l.* 7*s.* in $5\frac{1}{2}$ years?

(22) At what rate per cent. will 560*l.* 12*s.* $3\frac{1}{2}$*d.* gain 235*l.* 9*s.* 4*d.* in 7 years?

Case 9. When the principal, amount, and time, are given, to find the rate.

RULE.

Take the difference between the amount and principal, and divide it by the product of the principal and time: the quote is the rate.

Thus, THEOREM 8. $\frac{a-p}{pt}=r$.

EXAMPLES.

(23) At what rate per cent. will 284*l.* 10*s.* amount to 354*l.* 4*s.* $0\frac{1}{2}$*d.* in 7 years?

(24) At what rate per cent. will 378*l.* 18*s.* amount to 500*l.* 9*s.* $3\frac{1}{2}$*d.* in 6 years?

(25) At what rate per cent. will 672*l.* 5*s.* amount to 847*l.* 17*s.* 6*d.* in $5\frac{3}{4}$ years?

LXII. Of ANNUITIES, PENSIONS, &c. in ARREARS, at SIMPLE INTEREST.

AN annuity is a yearly income arising from money &c and is either paid for a term of years, or upon a life

Annuities or pensions are said to be in arrears when they are payable or due either yearly, half-yearly, or quarterly, and are unpaid for any number of payments.

Here U represents the annuity, pension, or yearly rent, A, T, R, as before.

Case 10. When U, R, and T are given, to find A.

THEOREM 9. $\frac{ttu-tu}{2} \times r : + tu = A.$

When the annuity, &c. is to be paid half-yearly, or quarterly, then for half-yearly payments take half the ratio, half the annuity, &c. and twice the number of years; and for quarterly payments take a fourth part of the ratio, a fourth part of the annuity, and four times the number of years; which work with as per theorem.

EXAMPLES.

(26) If 250*l.* yearly rent, pension, &c. be forborne or unpaid 6 years, what will it amount to in that time, at 3 per cent. for each payment, as it becomes due?

(27) If a salary of 250*l.* payable every half-year remain unpaid for 6 years, what would it amount to in that time, at 3 per cent. per annum?

(28) If a salary of 250*l.* payable every quarter, were left unpaid for 6 years, what would it amount to in that time, at 3 per cent. per annum?

It may be observed, by comparing the answers of the three last Examples, that the half-yearly payment is more advantageous than the yearly one, and also the quarterly more than the half-yearly.

Case 11. When A, R, and T are given, to find U.

THEOREM 10. $\frac{2a}{ttr-tr+2t} = U.$

When the payments are half-yearly, take $4a$; if quarterly, $8a$; and proceed with the ratio and time as before.

EXAMPLES.

(29) If a salary, payable yearly, amount to 1612*l.* 10*s.* in 6 years, at 3 per cent. what is the salary?

(30) The amount of a salary payable half-yearly, for 6 years, at 3 per cent. is 1623*l.* 15*s.* What is the salary?

(31) If the amount of an annuity, payable quarterly, be 1629*l.* 7*s.* 6*d.* for 6 years, at 6 per cent. what is the annuity?

Case 12. When U, A, and T are given, to find R.

THEOREM 11. $\frac{2a-2ut}{utt-ut}=\mathrm{R}.$

When the payments are half-yearly, take $4a-4ut$ for a dividend; if quarterly, take $8a-ut$, and proceed with the annuity and time as is mentioned in theorem 9.

EXAMPLES.

(32) If a salary of 250*l.* per annum amount to 1612*l.* 10*s.* in 6 years, what is the rate per cent.?

(33) If a salary of 250*l.* per annum, payable half-yearly, amount to 1623*l.* in 6 years, what is the rate per cent.?

(34) Suppose a pension of 250*l.* per annum, payable quarterly, amount to 1629*l.* 7*s.* 6*d.* in 6 years, what is the rate per cent.?

Case 13. When U, A, and R are given, to find T.

THEOREM 12. First $\frac{2}{r}-1=\mathrm{X}$. Then $\sqrt{\frac{2a}{ur}+\frac{xx}{4}}:\frac{x}{2}=\mathrm{T}.$

When the payments are half-yearly or quarterly, proceed with the ratio and annuity as mentioned before, and T will be equal to those half-yearly or quarterly payments.

EXAMPLES.

(35) In what time will a salary of 250*l.* per annum amount to 1612*l.* 10*s.* at 3 per cent.?

(36) If an annuity of 250*l.* per annum, payable half-yearly,

amount to 1623*l.* 15*s.* at 3 per cent. what time were the payments forborne?

(37) If an annuity of 250*l.* per annum, payable quarterly, amount to 1629*l.* 7*s.* 6*d.* at 3 per cent. what was the time of forbearance?

LXIII. PRESENT WORTH of ANNUITIES, &c.

HERE P represents the present worth, U, T, and R, as before.

Case 14. When U, T, and R are given, to find P.

THEOREM 13. $\frac{\overline{ttr-tr+2t}}{\overline{2tr+2}} : \times U = P.$

The same is to be observed here for half-yearly and quarterly payments as before mentioned.

EXAMPLES.

(38) What is the present worth of 250*l.* per annum, to continue 6 years?

(39) What is the pension of 250*l.* per annum worth in ready money, payable half-yearly, at 3 per cent. for 6 years?

(40) What is the present worth of 250*l.* payable quarterly, for 6 years, at 3 per cent.?

Case 15. When P, T, and R are given, to find U.

THEOREM 14. $\frac{tr+1}{\overline{ttr-tr+2t}} : \times 2p = U.$

When the payments are half-yearly, multiply by $4p$; for quarterly, by $8p$; and proceed with t and r as before directed.

EXAMPLES.

(41) What annuity is that which, for 6 years' continuance, produces 1366*l.* 10*s.* 6*d.* present worth, at 3 per cent.?

(42) There is an annuity, payable half-yearly, for 6 years to come; what is the yearly income, when the present worth, at 3 per cent. is 1376*l.* 5*s.*?

(43) There is an annuity, payable quarterly, for 6 years to come; what is the yearly income, when the present worth is 1380*l.* 17*s.* 6*d.* at 3 per cent.?

Case 16. When U, P, and T are given, to find R.

THEOREM 15. $\frac{ut-p\times 2}{2pt+ut-utt}=\mathrm{R}.$

When the payments are half-yearly or quarterly, proceed with the annuity and time as before directed; and the quotient will be the answer accordingly; i. e. if for half-yearly, the quotient will be half the ratio, and if for quarterly, a fourth part of the ratio.

EXAMPLES.

(44) At what rate per cent. will an annuity of 250*l.* per annum, to continue 6 years, produce the present worth of 1366*l.* 10*s.* 6*d.*?

(45) If an annuity of 250*l.* per annum, payable half-yearly, having 6 years to come, be sold for 1376*l.* 5*s.* what is the rate per cent.?

(46) At what rate per cent. will an annuity of 250*l.* per annum, payable quarterly, to continue 6 years, produce 1380*l.* 17*s.* 6*d.* for the present worth?

Case 17. When U, P, and R are given, to find T.

THEOREM 16. First $\overline{\frac{2}{r}\quad\frac{2p}{a}}-1=x.$

Then $\sqrt{\frac{2p}{ur}+\frac{xx}{4}}-\frac{x}{2}=t.$

When the payments are half-yearly, or quarterly, proceed with the annuity and ratio as before directed, and the quotient will be the number of payments.

EXAMPLES.

(47) If an annuity of 250*l.* per annum produce 1366*l.* 10*s.* 6*d.* for the present worth at 3 per cent., what is the time of its continuance?

(48) An annuity of 250*l.* per annum, payable half-yearly, is sold for 1376*l.* 5*s.* at 3 per cent. I desire to know the number of payments and time to come.

(49) Suppose a lease of a house of 250*l.* per annum, payable quarterly, be sold for 1380*l.* 17*s.* 6*d.* at 3 per cent. I demand the number of payments, and time to come.

LXIV. ANNUITIES, &c. taken in REVERSION.

Case 18. To find the present worth of an annuity taken in Reversion.

RULE.	THEOREM 17.
1. Find the present worth of the yearly sum at the given rate, and for the time of its continuance.	1. Thus $\frac{\overline{ttr-tr+2t}}{2tr+2}: \times U = P.$
2. Change P into A, and find what principal being put to interest will amount to A at the same rate, and for the time to come, before the annuity, &c. commences.	2. Thus $\frac{a}{tr+1} = P.$

EXAMPLES.

(50) What is the present worth of 250*l.* per annum, to continue 6 years, but not to commence until the end of 4 years, allowing 3 per cent. to the purchaser?

(51) What is the present worth of a lease of 80*l.* per annum, to continue $7\frac{1}{2}$ years, but not to commence until the end of 5 years, allowing $4\frac{1}{2}$ per cent. to the purchaser?

(52) There is a legacy of 40*l.* per annum, for 10 years, left to a person of 14 years of age; the time of payment is not to commence till the said person's age be 21; but he, wanting a sum of money, is inclined to sell the same at 5 per cent. I demand the present worth.

Case 19. To find the yearly income of an annuity, &c. in reversion.

RULE.

1. Find the amount of the present worth at the given rate, and for the time of its continuance.
2. Change A into P, and find what annuity being sold will produce P at the same rate, and for the time of its continuance.

Thus, THEOREM 18.

$$ptr+p=A.$$

Thus

$$\frac{tr+1}{ttt-tr+2t}:2p=U.$$

EXAMPLES.

(53) A person having an annuity left him for 6 years and 5 months, but not to commence till the end of 4 years, has disposed of it for the present payment of 1220*l.* 2*s.* 2¾*d.* allowing 3 per cent. to the purchaser. What is the yearly income?

(54) There is a lease of a house taken for 7½ years, but not to commence until the end of 5 years: the lessee would sell the same for 419*l.* 15*s.* 1¾*d.* present payment, allowing 4½ per cent. to the purchaser. What is the yearly rent?

(55) There is a legacy of a certain rate per annum, for 10 years, left to a person of 14 years of age; but the time of payment is not to commence till the said person's age be 21 years; but he, wanting a sum of money, sold it for 241*l.* 19*s.* 6*d.* allowing 5 per cent. to the buyer. I demand the yearly rate.

LXV. REBATE or DISCOUNT.

HERE S represents the sum to be discounted, P the present worth, T and R as before.

Case 20. When S, T, and R are given, to find P.

THEOREM 19. $\frac{s}{tr\times 1}=P.$

(56) What is the present worth of 150*l.* due 9 months hence, at 5 per cent.?

(57) What is the present worth of 1000*l.* due at 5 months, at $4\frac{1}{2}$ per cent.?

(58) What is the discount of 9342*l.* at 4 per cent, for 10 months?

Case 21. When P, T, and R are given, to find S.

THEOREM 20. $ptr+p=S$.

EXAMPLES.

(59) Suppose I receive 144*l.* 11*s.* $6\frac{3}{4}$*d.* now, for a sum of money due 9 months hence, allowing 5 per cent. for present payment, I demand the sum that was due at first.

(60) If the present worth of a sum of money due 5 months hence, allowing $4\frac{1}{2}$ per cent. be 981*l.* 10*s.* 5*d.* what was the sum first due?

(61) A person paid 9111*l.* 3*s.* $8\frac{1}{2}$*d.* for a debt due 10 months hence, he being allowed 4 per cent. for the discount. How much was the debt?

Case 22. When S, P, and R are given, to find T.

THEOREM 21. $\frac{s-p}{pr}=T$.

EXAMPLES.

(62) The present worth of 150*l.* due for a certain time to come, is 144*l.* 12*s.* $6\frac{1}{4}$*d.* at 5 per cent. I demand in what time the first sum should have been paid, if no rebate had been made.

(63) A person receives 981*l.* 10*s.* 5*d.* for 1000*l.* due at a certain time to come, allowing $4\frac{1}{2}$ per cent. discount. I desire to know in what time the debt should have been discharged without any rebate.

(64) I have received 9111*l.* 3*s.* $8\frac{1}{2}$*d.* for a legacy of 9342*l.* allowing the executor 4 per cent. I demand when the legacy was payable without rebate.

Case 23. When S, P, and T are given, to find R.

THEOREM 22. $\frac{s-p}{pt}=R$.

EXAMPLES.

(65) At what rate per cent. will 150*l.* payable 9 months hence, produce 144*l.* 11*s.* $6\frac{3}{4}$*d.* for the present payment?

(66) At what rate per cent. will 1000*l.* payable at 5 months hence, produce 981*l.* 10*s.* 5*d.* for the present payment?

(67) At what rate per cent. will 9342*l.* payable 10 months hence, produce 9111*l.* 3*s.* $8\frac{1}{2}$*d.* for the present payment?

LXVI. EQUATION of PAYMENTS.

TO find the equated time for the payment of any sum of money, due at several times.

RULE.

1. Find the present worth of each payment for its respective time. | Thus, THEOREM 23. $\frac{s}{tr+1}=P.$

2. Add all the present worths together, and call that sum P; then will $s-p=D$, the rebate.

3. Add $\frac{d}{pr}=E$, the true equated time.

EXAMPLES.

(68) D. owes C. 1400*l.* which was to have been paid as follows: 400*l.* down; 500*l.* at the end of 6 months; 250*l.* at the end of 8 months, and the rest at the end of 10 months; but they agree to have but one payment of the whole, rebate at $3\frac{1}{3}$ per cent. The true equated time is demanded.

(69) In what time will the interest of 49*l.* 3*s.* equal the proceed of 12*l.* 6*s.* of use 47 days, at any rate of interest?

(70) Put out 384*l.* to interest, and in 8¾ years there were 542*l.* 8*s.* found to be due. What rate of interest could then be implied?

LXVII. COMPOUND INTEREST.

The letters, made use of here are,

A, the amount.
P, the principal.
T, the time.
R, the amount of 1*l.* for 1 year, at any given rate which is found by the following proportion.

Thus,

$$\text{As}\begin{cases}100 : 105 :: 1 : 1{,}05 = \text{R, at 5 per cent.}\\ 100 : 106 :: 1 : 1{,}06 = \text{R, at 6 per cent., \&c.}\end{cases}$$

The construction of the first table following, showing the amount of 1*l.* for any number of years under 31, at 3, 3½, 4, 4½, and 5 per cent.

Thus the amount of 1*l.* for 2 years, at 5 per cent. compound interest, will be $1{,}05 \times 1{,}05 = 1{,}025$.

Also, $1{,}05 \times 1{,}05 \times 1{,}05 = 1{,}157625$ = the amount of 1*l.* for 3 years, at 5 per cent.

And the construction of the second table is by the continual multiplication of the amount of 1*l.* for a day; the amount of 1*l.* for a day being the root of its amount for a year, extracted to the 365th power.

The amount of 1*l.* for a day at 5 per cent. is 1,0001336, its amount for 2 days will be $1{,}0001336 \times 1{,}0001336$, $= 1{,}0002672$, &c. and $1{,}0001336 \times 1{,}0001336 \times 1{,}0001336 = 1{,}0004011$, the amount of 1*l.* at 5 per cent. for 3 days, compound interest.

TABLE I.

The Amount of one Pound for Years.

Years.	3 per Cent.	3½ per Cent.	4 per Cent.	4½ per Cent	5 per Cent.
1	1,0300000	1,0350000	1,0400000	1,0450000	1,0500000
2	1,0609000	1,0712259	1,0816000	1,0920250	1,1025000
3	1,0927270	1,1087178	1,1248640	1,1411661	1,1576250
4	1,1255088	1,1475230	1,1698586	1,1925186	1,2155063
5	1,1592740	1,1876863	1,2166529	1,2461816	1,2762816
6	1,1948523	1,2292553	1,2653190	1,3022601	1,3400956
7	1,2298733	1,2722792	1,3159318	1,3608618	1,4071004
8	1,2667700	1,3168090	1,3685691	1,4221006	1,4744554
9	1,3047731	1,3628973	1,4223118	1,4860251	1,5513282
10	1,3439163	1,4105987	1,4862443	1,5529694	1,6288946
11	1,3842338	1,4599697	1,5394541	1,6228530	1,7103393
12	1,4257608	1,5110686	1,6010322	1,6958814	1,7958563
13	1,4685337	1,5639560	1,6650735	1,7721961	1,8856491
14	1,5125897	1,6186045	1,7316764	1,8519449	1,9799376
15	1,5579674	1,6753488	1,8009435	1,9352824	2,0789282
16	1,6017064	1,7339860	1,8729812	2,0223901	2,1828746
17	1,6528476	1,7946755	1,9479005	2,1183768	2,2920183
18	1,7024330	1,8574892	2,0258165	2,2308478	2,4066192
19	1,7535060	1,9225013	2,1068492	2,3078603	2,5269502
20	1,8061112	1,9897898	2,1911231	2,4117149	2,6532977
21	1,8602945	2,0594314	2,2787681	2,5202411	2,7859626
22	1,9161034	2,1315115	2,3699188	2·6336520	2,9252607
23	1,9735865	2,2061144	2,4647155	2,7521663	3,0715238
24	2,0327941	2,2833284	2,5633042	2,8760138	3,2251000
25	2,0937779	2,3632449	2,6658363	3,0054344	3,3863549
26	2,1565912	2,4459583	2,7724697	3,1406709	3,5546527
27	2,2112890	2,5315671	2,8833685	3,2820095	3,7334563
28	2,2879276	2,6201719	2,9987033	3,4296999	3,9201291
29	2,3565655	2,7118779	3,1186514	3,5840364	4,1161336
30	2,4272624	2,8067937	3,2433975	3,7453181	4,3216424

TABLE II.

The Amount of one Pound for Days.

Days	3 per Cent.	3½ per Cent.	4 per Cent.	4½ per Cent.	5 per Cent.
1	1,0000809	1,0000942	1,0001074	1,0001206	1,0001330
2	1,0001619	1,0001885	1,0002149	1,0002412	1,0002973
3	1,0002429	1,0002827	1,0003224	1,0003618	1,0004011
4	1,0003240	1,0003770	1,0004299	1,0004824	1,0005348
5	1,0004050	1,0004713	1,0005374	1,0006031	1,0006685
6	1,0004860	1,0005656	1,0006449	1,0007238	1,0008025
7	1,0005670	1,0006600	1,0007524	1,0008445	1,0009361
8	1,0006480	1,0007542	1,0008600	1,0009652	1,0010699
9	1,0007291	1,0008486	1,0009675	1,0010859	1,0012037
10	1,0008101	1,0009429	1,0010751	1,0012066	1,0013376
20	1,0016209	1,0011867	1,0021512	1,0024148	1,0026770
30	1,0024318	1,0028315	1,0032286	1,0036243	1,0040182
40	1,0032445	1,0037771	1,0043074	1,0048354	1,0053611
50	1,0040573	1,0047236	1,0053871	1,0060479	1,0067059
60	1,0048708	1,0056710	1,0064680	1,0072618	1,0080525
70	1,0056849	1,0066193	1,0075501	1,0084773	1,0094009
80	1,0064996	1,0075685	1,0086333	1,0096942	1,0107511
90	1,0073151	1,0085186	1,0097177	1,0109125	1,0121031
100	1,0081311	1,0094696	1,0108033	1,0121324	1,0134569
110	1,0089479	1,0104214	1,0118900	1,0133537	1,0148125
120	1,0097653	1,0113742	1,0129779	1,0145765	1,0161699
130	1,0105834	1,0123279	1,0140670	1,0158007	1,0175291
140	1,0114021	1,0132825	1,0151572	1,0170265	1,0188932
150	1,0122215	1,0142379	1,0162487	1,0182537	1,0202531
160	1,0130415	1,0151943	1,0173412	1,0194824	1,0216178
170	1,0138623	1,0161516	1,0184350	1,0207126	1,0229843
180	1,0146837	1,0171098	1,0195299	1,0219442	1,0243527
190	1,0155057	1,0180689	1,0206261	1,0231774	1,0257228
200	1,0163284	1,0190288	1,0217233	1,0244120	1,0270949
210	1,0171518	1,0199897	1,0228218	1,0256481	1,0284687
220	1,0179759	1,0209315	1,0239215	1,0268858	1,0298444
230	1,0188006	1,0219142	1,0250223	1,0281249	1,0312219
240	1,0196260	1,0228778	1,0261243	1,0293655	1,0326013
250	1,0204520	1,0238424	1,0272275	1,0306076	1,0339825

Case 1. When P, T, and R are given, to find A.

* THEOREM 1. $p \times r^t = A$.

By the foregoing tables, thus: Multiply the principal by the tabular number for the given time and rate; and the product will be the amount required.

If the amount be required for any number of years or days that are not in the tables, then observe this rule. Divide the given number of years or days into such numbers as are in the table; then multiply the amounts answering to each, into each other, continually, and the product by the principal, which will be the amount required.

EXAMPLES.

(1) What will 246*l.* 10*s.* amount to in 7 years, at 5 per cent. per annum?

(2) What will 500*l.* amount to in 30 years and 30 days, at $4\frac{1}{2}$ per cent.?

(3) What is the amount of 523*l.* in 5 years and 194 days, at 5 per cent.?

Case 2. When A, R, and T are given, to find P.

THEOREM 2. $\frac{A}{R^t} = P$.

By the foregoing tables, thus: Divide the amount by the tabular number for the given time and rate, and the quotient will be the principal required.

EXAMPLES.

(4) What principal, or sum of money, must be put out to

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* The most convenient way of giving the four Theorems in Compound Interest is by Logarithms, as follows:

1. $t \times \log.r + \log.p = \log.A.$
2. $\log.a - t \times \log.r = \log.P.$
3. $\dfrac{\log.a - \log.p.}{\log.r.} = T.$
4. $\dfrac{\log.a - \log p.}{t} = \log.R.$

raise a stock of 243*l.* 2*s.* $\frac{1}{4}$*d.* in 4 years, at 5 per cent. per annum?

(5) What principal, being put to interest, will amount to 346*l.* 17*s.* in 7 years, at 5 per cent. per annum?

(6) What principal, being put to interest for 30 years at $4\frac{1}{2}$ per cent. per ann. will amount to 1872*l.* 13*s.* 2*d.*?

(7) What principal, being put to interest for 5 years and 194 days, will amount to 685*l.* $7\frac{1}{2}$*d.* at 5 per cent. per annum?

Case 3. When P, A, and R are given, to find T.

THEOREM 3. $\frac{p}{a}=R^t$. { Which being continually divided by r, till nothing remains, the number of those divisions will be equal to T, the time.

By the foregoing tables, thus: Divide the amount by the principal, and the quotient will be the amount of 1*l.* at the given rate, which will be found under the rate, even with the time required.

EXAMPLES.

(8) In what time will 246*l.* 10*s.* amount to 346*l.* 17*s.* at 5 per cent. per annum?

(9) In what time will 530*l.* amount to 1872*l.* 13*s.* 2*d.* at $4\frac{1}{2}$ per cent. per annum?

(10) In what time will 510*l.* amount to 685*l.* $7\frac{1}{2}$*d.* at 5 per cent. per annum?

Case 4. When A, P, and T are given, to find R.

THEOREM 4. $\frac{a}{p}=R^t$. { Which being extracted by the rules of Extraction (the time given to the question showing the power) will give the rate.

By the foregoing tables, thus: Proceed as with the last, and even with the given time is under the rate required.

EXAMPLES.

(11) At what rate per cent. will 246*l.* 10*s.* amount to 346*l.* 17*s.* in 7 years?

(12) At what rate per cent. will 500*l.* amount to 1872*l.* 13*s.* 2*d.* in 30 years?

(13) At what rate will 523*l.* amount to 685*l.* 7½*d.* in 5 years and 195 days?

LXVIII. ANNUITIES, PENSIONS, in ARREARS.

HERE U represents the annuity, pension, or yearly rent, A, R, and T, as before.

The 3d Table shows the amount of 1*l.* annuity for any number of years, under 31, at 3, 3½, 4, 4½, and 5 per cent. and is thus constructed: Take the first year's amount, which is 1*l.*, multiply by 1,05+1=2,05=2d year's amount; which also multiply by 1,05+1=3,1525=3d year's amount.

The 4th Table shows the present worth of 1*l.* due at any number of years, to commence under 31, rebate at 3, 3½, 4, 4½, and 5 per cent. and is made thus: 1÷1,05=952381 first year's present worth, and ,952381÷1,05=9070295=2d year's, and ,90703÷1,05=,8638376, the third year's present worth, &c.

TABLE III.

The Amount of One Pound per Annum, or Annuity, for Years.

Years.	3 per Cent.	3½ per Cent.	4 per Cent.	4½ per Cent.	5 per Cent.
1	1,0000000	1,0000000	1,0000000	1,0000000	1,0000000
2	2,0300000	2,0350000	2,0400000	2,0450000	2,0500000
3	3,0909000	3,1062250	3,1216000	3,1370250	3,1525000
4	4,1836270	4,2149429	4,2464640	4,2781911	4,3101250
5	5,3091358	5,3625659	5,4707097	5,4707097	5,5256312
6	6,4684099	6,5501522	6,6329755	6,7168917	6,8019128
7	7,6624622	7,7794075	7,8982943	7,9191518	8,1420084
8	8,8923360	9,0516866	9,2142263	9,3800136	9,5493643
9	10,159106	10,3684958	10,5827953	10,8021142	11,026787
10	11,4638793	11,7313931	12,026107	12,2882094	12,5771925
11	12,8077957	13,1419919	13,4863514	13,8411788	14,2067871
12	14,1920296	14,6019616	15,0258055	15,4640319	15,9171265
13	15,6177904	16,1130303	16,6262397	17,1599133	17,7129828
14	17,0863242	17,6769864	18,2919112	18,9321094	19,5986320
15	18,5989139	19,2956809	20,0235876	20,7840543	21,5785636
16	20,1568813	20,9710297	21,8245311	22,7193367	23,6574918
17	21,7615877	22,7050158	23,6975124	24,7417069	25,8403664
18	23,4144354	24,4996912	25,6454129	26,8550837	28,1323841
19	25,1168684	26,3571805	27,6712294	29,0635625	30,5390039
20	26,8703745	28,2796818	29,7780786	31,3714228	33,0659547
21	28,6764857	30,2694707	31,9692017	33,7831368	35,7192518
22	30,5367803	32,3289022	34,2479698	36,3033779	38,5052144
23	32,4529837	34,4604137	36,6178886	38,9370299	41,4304651
24	34,4264702	36,6665282	39,0826041	41,6891963	44,5019989
25	36,4592643	38,9498567	41,6459083	44,5652101	47,7270988
26	38,5530422	41,3131017	44,3117440	47,5706446	51,1134538
27	40,7096335	43,7590602	47,0842144	50,7113236	54,6691265
28	42,9309225	46,2906273	49,9675830	53,9933332	58,4025828
29	45,2188502	48,9107993	52,9662863	57,4230332	62,3227119
30	47,5754157	51,6226773	56,0849277	61,0070607	66,4388475

TABLE IV.

The Present Worth of one Pound for Years.

Years.	3 per Cent.	3½ per Cent.	4 per Cent.	4½ per Cent.	5 per Cent.
1	,9708738	,9661836	,9615385	,9569378	,9523809
2	,9425959	,9331507	,9245562	,9157299	,9070295
3	,9151417	,9019427	,8889964	,8762966	,8638376
4	,8884870	,8714422	,8548042	,8385613	,8227025
5	,8626088	,8419732	,8219271	,8024511	,7835262
6	,8374843	,8135006	,7903145	,7678957	,7462154
7	,8130915	,7859910	,7599178	,7348285	,7106818
8	,7894092	,7594116	,7306902	,7031851	,6768394
9	,7664167	,7337310	,7025867	,6729044	,6446089
10	,7440939	,7809183	,6755642	,6439277	,6139133
11	,7224213	,6849457	,6495809	,6161988	,5846703
12	,7013799	,6617833	,6245971	,5896639	,5568374
13	,6809513	,6394041	,6005741	,5642716	,5303214
14	,6611178	,6177818	,5774751	,5399729	,5050679
15	,6418619	,5968906	,5552645	,5167204	,4810171
16	,6231669	,5767059	,5339082	,4944693	,4581115
17	,6050164	,7572038	,5133783	,4731764	,4362967
18	,5873846	,5383611	,4936281	,4528004	,4155207
19	,5702960	,5201577	,4746424	,4333018	,3957340
20	,5536758	,5025659	,4563870	,4146429	,3768895
21	,5375493	,4855709	,4388336	,3967874	,3489424
22	,5218925	,4691506	,4219554	,3797009	,3418479
23	,5066917	,4532856	,4057263	,3633501	,3255713
24	,4919337	,4379571	,3901215	,3477035	,3100639
25	,4776056	,4231470	,3751168	,3327306	,2953028
26	,4636947	,4088378	,3606892	,3184025	,2812407
27	,4501891	,3950123	,3468166	,3046914	,2678483
28	,4370768	,3816543	,3334775	,2915707	,2550936
29	,4243464	,3687482	,3206514	,2790150	,2429463
30	,4119868	,3562784	,3083187	,2670000	,2315775

Case 5. When U, T, and R are given, to find A.

THEOREM 5. $\frac{Ur-u}{r-1}=A.$

By Table III. thus: Multiply the annuity by the tabular number for the given time and rate, and the product will be the amount required.

EXAMPLES.

(14) What will an annuity of 70*l.* per annum (payable yearly) amount to in 4 years, at 4 per cent. per annum?

(15) If a salary of 100*l.* per annum, to be paid yearly, be forborne 6 years, or unpaid, at 5 per cent. per annum, what is the amount?

(16) A minor of 14 had an annuity left him of 70*l.* a-year, the proceed of which, by will, was to be put out, both principal and interest, yearly, as it fell due, at 3 per cent. till he should attain to 21 years of age; the utmost improvement being made of this part of his fortune, what had he then to receive?

(17) If an annuity of 30*l.* per annum, payable yearly, be omitted to be paid for 30 years, at 3½ per cent. what is the amount?

Case 6. When A, T, and R are given, to find U.

THEOREM 6. $\frac{ar-a}{r-1}=U.$

By Table III. thus: Divide the amount given by the tabular number for the given time and rate, and the quotient will be the annuity required.

EXAMPLES.

(18) What pension, being forborne 6 years, at 5 per cent. will amount to 680*l.* 3*s.* 9¼*d.* ,6288?

(19) What annuity will amount to 536*l.* 7*s.* 5½*d.* ,45984 in 7 years, at 3 per cent.?

(20) If the payment of an annuity, forborne for 30 years, amount to 1548*l.* 18*s.* 7¾*d.* at 3½ per cent. what is the annuity?

Case 7. When U, A, and R are given, to find T.

THEOREM 7. $\frac{\overline{ar+u-a}}{u}=R^t$. $\left\{\begin{array}{l}\text{Which proceed with as in}\\ \text{Theorem the 3d, which will}\\ \text{give T, the time.}\end{array}\right.$

By Table III. thus: Divide the amount by the annuity, and the quotient will be the amount of 1*l.* at the given rate, which will be found under the said rate even with the time required.

EXAMPLES.

(21) In what time will a salary of 100*l.* per annum amount to 680*l.* 3*s.* 9¾*d.* at 5 per cent.?

(22) In what time will an annuity of 70*l.* amount to 536*l.* 7*s.* ¾*d.* ,45984 at 3 per cent.?

(23) In what time will 30*l.* per annum amount to 1548*l.* 13*s.* 7¾*d.* ,45984 at 3½ per cent.?

Case 8. When A, U, and T are given, to find R.

THEOREM 8. $\frac{ar}{u}-rt=\frac{a-u}{u}R^t$.

This being a very high equation, it requires the assistance of Algebra to determine R by the Theorem.

By Table III. thus: Proceed as in the last Rule, which will give the annuity that 1*l.* will purchase for the given time, which will stand even with the said time, and under the rate required.

EXAMPLES.

(24) At what rate per cent. per annum would a salary of 100*l.* per annum amount to 686*l.* 3*s.* 9¾*d.* ,6288 in 6 years?

(25) At what rate per cent. per annum will an annuity of 70*l.* per annum amount to 536*l.* 7*s.* 5¾*d.* ,45984 in 7 years?

(26) At what rate per cent. per annum will a salary of 30*l.* per annum amount to 1548*l.* 13*s.* 7¾*d.* ,0264 in 30 years?

LXIX. PRESENT WORTH of ANNUITIES, &c.

THE 5th Table shows the present worth of 1*l.* annuity for any number of years under 31, at 3, 3½, 4, 4½, and 5 per

cent. and is made thus: divide 1 by 1,05=,95238, the present worth for the first year, which ÷1,05=,90703 added to the first year's present worth, =1,85941, the second year's present worth: again ,90703÷1,05, and the quotient added to 1,85948=2,72324= third year's present worth, at 5 per cent., &c.

The 6th Table shows the annuity which 1*l.* will purchase for any number of years under 31, at 3, 3½, 4, 4½, and 5 per cent. and is constructed by finding the present worth of 1*l.* per annum in the fifth Table, at the assigned rate and time: and dividing unity or 1 thereby, the quotient will be the annuity that 1*l.* will purchase at the same rate for the same time.

TABLE V.

The present Worth of One Pound per Annum, or Annuity, for Years.

Years.	3 per Cent.	3½ per Cent.	4 per Cent.	4½ per Cent	5 per Cent.
1	0,9708738	0,9661836	0,9615385	0,9569378	0,9523809
2	1,9124697	1,8996943	1,8860947	1,8726678	1,8594104
3	2,8286114	2,8016370	2,7750910	2,7489644	2,7232480
4	3,7170984	3,6730792	3,6298952	3,5875257	3,5459505
5	4,5797072	4,5150524	4,4518223	4,3899767	4,3294767
6	5,4171914	5,3285530	5,2421369	5,1578725	5,0756921
7	6,2302829	6,1145439	6,0020547	5,8927009	5,7863737
8	7,0196922	6,8739555	6,7327448	6,5958861	6,4632128
9	7,7861039	7,6076865	7,4353314	7,2687905	7,1078217
10	8,5302028	8,3169053	8,1108955	7,9127182	7,7217349
11	9,5256241	9,0015510	8,7604763	8,3289169	8,3064142
12	9,9540040	9,6633843	9,3850733	9,1185806	8,8632516
13	10,6349553	10,3027585	9,9856478	9,6828524	9,3935730
14	11,2960734	10,9205203	10,5631224	10,2228253	9,8986409
15	11,9379351	11,5174109	11,1183868	10,7395457	10,3796380
16	12,5611020	12,0941168	11,6522949	11,2340151	10,8377695
17	13,1661885	12,6513206	12,1656680	11,7071914	11,2740622
18	13,7535131	13,1896817	12,6592961	12,1599918	11,6895869
19	14,3237991	13,7098374	13,1339385	12,5939936	12,0853208
20	14,8774748	14,2124033	13,5903253	13,0099365	12,4622103
21	15,4150241	14,6979742	14,0291589	13,4047239	12,8211527
22	15,9369166	15,1671248	14,4511142	13,7844248	13,1630026
23	16,4436084	15,6204105	14,8568405	14,1477749	13,4885739
24	16,9355421	16,0583676	15 2469619	14,4954784	13,7986418
25	17,4131477	16,4815146	15,6220787	14,8220089	14,0939445
26	17,8768420	16,3903503	15,9827678	15,1466115	14,3751853
27	18,3270315	17,2853645	16,3295844	15,4513018	14,6430236
28	18,7641082	17,6670188	16,6630618	15,7428735	14,8981272
29	19,8184546	18,0357670	16,9837132	16,0218885	15,1410735
30	19,6004413	18,3920454	17,2920318	16,2888885	15,3724510

TABLE VI.

The Annuity which One Pound will purchase for any Number of Years.

Years.	3 per Cent.	3½ per Cent.	4 per Cent.	4½ per Cent.	5 per Cent.
1	1,0300000	1,0350000	1,0400000	1,0450000	1,0500000
2	,5226108	,5264005	,5301961	,5339976	,5378049
3	,3535304	,3569342	,3603485	,3637734	,3672086
4	,2690271	,2722511	,2754901	,2785437	,2820118
5	,2183546	,2214814	,2246271	,2277916	,2309748
6	,1845775	,1876682	,1907619	,1938784	,1970175
7	,1605064	,1635445	,1606096	,1697015	,1728198
8	,1424564	,1454767	,1485279	,1516097	,1547218
9	,1284339	,1314460	,1344930	,1375745	,1406001
10	,1172305	,1202414	,1232909	,1263788	,1295046
11	,1080975	,1110920	,1141490	,1172482	,1203889
12	,1004621	,1034840	,1065512	,1096662	,1128254
13	,0940295	,0970616	,1001437	,1032754	,1064558
14	,0885263	,0915707	,0946690	,0978203	,1016240
15	,0837666	,0868251	,0899411	,0931133	,0963423
16	,0796109	,0826848	,0858200	,0890154	,0922699
17	,0759525	,0790431	,0821985	,0854176	,0886991
18	,0727087	,0758168	,0789933	,0822369	,0855462
19	,0698139	,0729403	,0791386	,0794073	,0827450
20	,0672157	,0703611	,0735818	,0768761	,0802426
21	,0548718	,0680366	,0712801	,0746006	,0779961
22	,0627474	,0659321	,0691988	,0725457	,0759705
23	,0608139	,0640188	,0673091	,0706825	,0741368
24	,0590474	,0622728	,0655868	,0689870	,0724709
25	,0574279	,0606740	,0640120	,0674890	,0709525
26	,0559383	,0592054	,0625674	,0660210	,0695643
27	,0545642	,0578524	,0612385	,0647195	,0683919
28	,0532932	,0566029	,0600130	,0635208	,0671225
29	,0521147	,0554454	,0588799	,0624146	,0660455
30	,0510193	,0543713	,0578301	,0618903	,0650614

Case 9. When U, R, and T are given, to find P.

THEOREM 9. $u - \frac{u}{r} : \div r - 1 = P.$

By Table V. thus: multiply the tabular number for the given time and rate by the annuity, &c. the product will be the present worth required.

EXAMPLES.

(27) What is the present worth of an annuity of 50*l.* per annum, to continue 8 years, at 5 per cent.

(28) What is the present worth of an annuity or yearly rent of 60*l.* to continue 6 years at 4 per cent.?

(29) What is the present worth of a pension of 1000*l.* per annum, for 21 years, at $4\frac{1}{2}$ per cent.?

Case 10. When P, T, and R are given, to find U.

THEOREM 10. $\frac{\overline{pr^t \times r - prt}}{r^t - 1} = U.$

By Table V. thus: divide the present worth by the tabular number for the given time and rate; and the quotient will be the annuity required.

Or by Table VI. thus: multiply the tabular number (for the given rate and time) by the present worth; and the product will be the annuity.

EXAMPLES.

(30) If the present worth of 323*l.* 3*s.* $2\frac{1}{2}$*d.* were required for a pension of 8 years to come, at 5 per cent. what was the pension?

(31) What annuity or yearly rent may be purchased for 314*l.* 10*s.* $6\frac{1}{4}$*d.* at 4 per cent. for 6 years?

(32) Suppose the present worth of a pension for 21 years, at $4\frac{1}{2}$ per cent. were 13404*l.* 14*s.* $5\frac{1}{2}$*d.* what was the pension?

Case 11. When U, P, and R are given, to find T.

THEOREM 11. $\frac{u}{p+u-pr}=R^t$. $\left\{\begin{array}{l}\text{Which proceed with as}\\ \text{in Theorem 3: it will}\\ \text{give T.}\end{array}\right.$

By Table V. thus: divide the present worth by the annuity, and the quotient will be the amount of 1*l.* at the given rate, which will be found under the said rate, even with the time required.

EXAMPLES.

(33) How long may one have a lease of 50*l.* yearly rent for 323*l.* 3*s.* $2\frac{1}{2}$*d.* allowing 5 per cent. to the purchaser?

(34) If an annuity of 60*l.* be purchased for 314*l.* 10*s.* $6\frac{1}{4}$*d.* at 4 per cent. what time ought it to continue?

(35) I demand what time a lease of 1000*l.* may be purchased for, when the present worth of 13404*l.* 14*s.* $5\frac{3}{4}$*d.* is made at $4\frac{1}{2}$ per cent.

Case 12. When U, P, and T are given, to find R.

THEOREM 12. $\frac{u}{p}=\frac{u}{p}R^t.+Rt.-Rt.+1.$

This being a very high equation, requires the assistance of Algebra to determine R.

By Table V. proceed as in the last Rule; and the quotient will be the amount of 1*l.* annuity for the given time, over which will be the rate required.

EXAMPLES.

(36) If an annuity of 50*l.* to continue for 8 years, be purchased for 325*l.* 3*s.* $2\frac{1}{2}$*d.* what rate of interest has the purchaser for his money?

(37) Suppose I give 314*l.* 10*s.* $6\frac{3}{4}$*d.* for an annuity of 60*l.* to continue 6 years, at what rate was interest allowed?

(38) If an annuity of 1000*l.* to continue 21 years, be purchased for 13404*l.* 14*s.* $5\frac{1}{2}$*d.* what rate of interest is the purchaser allowed for his money?

LXX. ANNUITIES, LEASES, &c. taken in REVERSION.

To find the present Worth of Annuities, &c. in Reversion.

RULE.

1. Find the present worth of the annuity, &c. at the given rate, and for the time of its continuance, by Theorem 9.
2. Change P into A; and find what principal being put to interest will amount to P, at the same rate and time to come before the annuity commences, by Theorem 2, which will give the present worth of the annuity.

Thus, THEO. 13.

$U - \frac{u}{r} : \div r - 1 = P.$

Thus $\frac{A}{Rt} = P.$

By Table V. thus: find the present value of 1*l.* per annum, at the given rate, both for the time being, and also for that and the time in reversion added together; then subtract the time in being from the other, and multiply the remainder by the annuity: the product will be the present worth required.

EXAMPLES.

(39) What is the present worth of a reversion of a lease of 80*l.* per annum, to continue 6 years, but not to commence till the end of 2 years, allowing 5 per cent. to the purchaser?

(40) What ought a person to give down in ready money for the reversion of 1000*l.* a-year, to continue 20 years, on a lease which cannot commence till 5 years are at an end, allowing the purchaser 5 per cent.?

To find the Yearly Income of an Annuity taken in Reversion.

RULE.

1. Find the amount of the present worth at the given rate, and for the time before the annuity commences, by Theorem 1.

THEOREM 14.

Thus, $pr = A.$

2. Change A. into P, and find what yearly rent, &c. being sold, will produce P. at the same rate, and for the time of its continuance, by Theorem 10.

THEOREM 15.

Thus, $\frac{\overline{ptr \times r - pr^t}}{r^t - 1} = \text{U}.$

By Table V. thus: proceed as in the last rule, and divide the present worth by the remainder: the quotient will be the annuity required.

EXAMPLES.

(41) What annuity, to be entered upon two years hence, and then to continue 6 years, may be purchased for 368*l.* 6*s.* 1*d.* ready money, allowing 5 per cent. to the purchaser?

(42) Suppose the present worth of a lease of an estate be 8132*l.* 14*s.* 8*d.* taken in reversion for 20 years, but not to commence till the end of 5 years, allowing 5 per cent. to the purchaser: what is the yearly rent?

LXXI. REBATE or DISCOUNT.

Here S represents the sum to be purchased.

When S, T, and R are given, to find P.

THEOREM 16. $\frac{st}{r^t} = \text{P}.$

By the 4th table, thus: Multiply the tabular number for the given time and rate, by the sum to be purchased; the product will be the present worth.

EXAMPLES.

(43) What is the present worth of 150*l.* payable 4 years hence, at 5 per cent.?

(44) What is the present worth of 743*l.* 4*s.* 9*d.* payable 6 years hence, at 4 per cent.?

When P, T, and R are given, to find S.

THEOREM 17. $p \times r^t = \text{S}.$

By Table IV. thus: Divide the present worth by the tabular number for the given time and rate; and the quotient will be the sum to be purchased.

EXAMPLES.

(45) If 123*l.* 8*s.* 1¼*d.* be received for a debt payable 4 years hence, and an allowance of 5 per cent. to the debtor for the present payment, what was the debt?

(46) If a sum of money, due 6 years hence, produce 587*l.* 7*s.* 9¾*d.* for the present payment, rebate being made at 4 per cent. I demand how much the debt was.

When S, P, and R are given, to find T.

THEOREM 18. $\frac{s}{p} = r^t$. { Which proceed with as in Theorem 3.

By Table IV. thus: Divide the principal by the sum to be purchased; and the quotient will be the present worth of 1*l.* at the given rate, which will be found under the rate, and even with the time required.

EXAMPLES.

(47) A person received 123*l.* 8*s.* 1¼*d.* for a debt of 150*l.*, rebate being made at 5 per cent. I demand in what time the debt was payable.

(48) The present payment of 587*l.* 7*s.* 9¾*d.* is made for a debt of 743*l.* 4*s.* 9*d.* rebate at 4 per cent. I demand when the debt was payable.

When S, P, and T are given, to find R.

THEOREM 19. $\frac{S}{p} = R^t$. { Which proceed with as in Theorem 4.

By Table IV. thus: Proceed as in the last rule, and the quotient will be the present worth of 1*l.* for the given time, which will be found even with the time, and under the rate required.

EXAMPLES.

(49) The present worth of 150*l.* payable 4 years hence, is 123*l.* 8*s.* 1¼*d.* At what rate per cent. is the rebate made?

(50) The sum of 743*l.* 4*s.* 9*d.* is payable in 6 years' time, and the present value of that sum is 587*l.* 7*s.* $0\frac{3}{4}$*d.* I demand at what rate per cent. the rebate must be made.

LXXII. PURCHASING FREEHOLD or REAL ESTATES

IS to find the present worth of an annuity, &c. to continue for ever.

When U and R are given, to find P.

THEOREM 20. $\frac{u}{r-1}=P.$

EXAMPLES.

(51) Suppose a freehold estate of 500*l.* per annum were to be sold; what is the worth, allowing 5 per cent. to the buyer?

(52) What is an estate of 25*l.* per annum, to continue for ever, worth in present money, allowing $4\frac{1}{2}$ per cent. to the buyer?

When P and U are given, to find R.

THEOREM 21. $\frac{P+u}{p}=R.$

EXAMPLES.

(53) Suppose one gave 10,000*l.* for a freehold estate of 500*l.* per annum, what rate per cent. has the purchaser for his money?

(54) If an estate of 25*l.* per annum be bought for 555*l.* 11*s.* $1\frac{1}{2}$*d.* what is the rate per cent.?

When P and R are given, to find U.

THEOREM 22. $\overline{P+r-1}=U.$

EXAMPLES.

(55) Suppose a person would lay out 10000*l.* on a freehold estate, and so as to be allowed 5 per cent. for his money, what must be the annual rent of such an estate?

(56) If a freehold estate be bought for 555*l.* 11*s.* 1¾*d.* and the allowance of 4½ per cent. be made to the buyer, what is the yearly rent?

LXXIII. PURCHASING FREEHOLD ESTATES in REVERSION.

To find the worth of a freehold estate in reversion.

RULE.	
1. Find the worth of the yearly rent, &c.	THEOREM 23. Thus $\frac{u}{r-1}=P$.
2. Change P into A, and find what principal being put to interest will amount to A, at the same rate, and for the time to come before the estate commences.	Thus $\frac{A}{R^t}=P$.

EXAMPLE.

(57) What is an estate of 500*l.* per annum worth in ready money, to continue for ever, but not to commence till the end of 4 years, allowing 5 per cent. to the purchaser?

To find the yearly rent of an estate taken in reversion.

RULE.	
1. Find the amount of the worth of the estate at the given rate, and the time before it commences.	THEOREM 24. $p \times r = A$.
2. Change A into P, and find what yearly rent being sold will produce P at the same rate.	$\frac{\overline{Pr \times r} - pr}{r} = U$.

EXAMPLE.

(58) A freehold estate is sold for 8227*l.* 1*s.* 4*d.* which does not commence till the end of 4 years, the buyer being allowed 5 per cent. for his money: I desire to know the yearly income.

QUESTIONS *for Exercise.*

(59) Held of a college 486*l.* 10*s.* a-year, on a reserved rent of 94*l.* Money being at 5 per cent. interest; what fine ought severally to be paid on a 7, a 14, and a 21 years' lease?

(60) Suppose I would add five years to a running lease of 15 years to come, the improved rent being 186*l.* 7*s.* 6*d.* per annum; what ought I to pay down in ready money for this favour, discounting 4 per cent.?

(61) A. has a term of 7 years in an estate of 50*l.* per annum; B. has a term of 14 years in the same estate; and C. has a farther term of 10 years after B. in the same estate. What is the present value of the several interests in the said estate?

(62) For a lease of certain profits for 7 years, A. offers to pay 150*l.* gratuity, and 300*l.* per annum; B. offers 400*l.* gratuity, and 250*l.* per annum; C. bids 650*l.* gratuity, and 200*l.* per annum; and D. offers 1800*l.* for the whole purchase, without any yearly rent. Query, which is the best offer? and what difference, computing at 4 per cent.?

(63) Value the lease of a house in tolerable repair, the rent 54*l.* 17*s.* a-year, the ground rent 7 guineas; 3 years of it only to come; the rent payable every 6 months; discount per compound interest on this kind of purchase at 10*l.* per cent.

(64) A fine for a lease of a tenement is settled at 153*l.* under a reserved rent of 16*l.* a-year. Now the tenant cannot conveniently pay more than 50*l.*; but for the 6 years to come of the term, is willing rather to pay an adequate rent, computing 10*l.* per cent. per compound interest. What ought that rent to be?

(65) Another lease for 7 years is agreed for at 250*l.* Fine, on the old rent 44*l.* a-year; but considering the contractor desires to reduce the rent to 20*l.* a-year, and pay a proper fine, computing as before, after the rate of 10*l.* a-year; to what must the fine be advanced?

(66) A son, previous to his marriage, is inclined to have 50*l.* a-year freehold settled on his family; and to have immediate possession of it, offers his father, in lieu, an annuity for his life, valued at 12 years'

purchase, discounting at 4 per cent. thereon: whereas he is content the estate should be valued at a discount of 3 per cent. and consequently will be worth 33⅓ years' purchase. Pray what had the father for his life?

(67) A gentleman took a college lease of 237*l.* a-year, for 21 years, and paid the full fine: the rent reserved was 10*l.* a-year; but when 4 years were elapsed, against his marriage, he renewed the lease, and filled up the 21 years. In 14 years after that, his wife dying, he again renewed it in favour of his daughter, then 7 years of age; and by the time she was 19, it was a third time renewed in order to her settlement. The question is, what money the society must have received from this family from first to last, allowing 5*l.* a-year discount on the fines.

PART IV.

LXXIV. MENSURATION.

GEOMETRICAL DEFINITIONS.

GEOMETRY contains the nature and properties of lines, angles, surfaces, and solids.

A point is that which has no parts nor magnitude.

A line is length conceived without breadth.

An angle is the mutual inclination of two lines which meet.

When a straight line, as CD (Fig. 4.), standing upon another, A B, makes the angles, A D C, and C D B, on each side equal to one another, each of these equal angles is called a right angle, and the dotted line, C D, is said to be perpendicular to the line A B.

An angle is commonly expressed by three letters; that placed at the angular point being always wrote in the middle, as A D C (Fig. 4.) denotes the angle *b*.

An obtuse angle is that which is greater than a right angle, as C A B (Fig. 3.).

An acute angle is that which is less than a right angle, as D C B (Fig. 4.).

Parallel lines are those of which every point of the one is at the same distance from the other, as the lines A B and C D (Fig. 2.).

A superficies, or surface, is an extension of two dimensions, viz. length and breadth.

A plane, or plane superficies, is that with which a right line may every way coincide.

A plane superficies receives several denominations, according to the number and positions of the lines by which it is terminated; as follow:

Fig. 1. A square is a right-angled equilateral parallelogram, whose four sides are equal, and its angles all right ones.

A quadrangle is a figure made by four straight lines.

Fig. 2. A parallelogram is a quadrangle whose opposite sides are parallel.

An oblong, or rectangle, is longer than broad; but its opposite sides are equal, and all its angles right ones.

A rhombus, or diamond figure, is a parallelogram whose sides are all equal, but its angles are not right angles.

Fig. 3. A rhomboides is an oblique-angled parallelogram, whose opposite sides and angles only are equal.

A triangle is a space included by three lines, and of consequence has three angles; for every rectilineal plane figure has as many angles as sides.

A right-angled triangle is that which has one right-angle, as Fig. in page 174.

Fig. 4. An equilateral triangle is that whose three sides are all equal to each other.

An isosceles triangle is that which has only two of its sides equal to one another.

A scalene triangle is that which has all its sides unequal.

An obtuse-angled triangle is that which has an obtuse angle.

An acute-angled triangle is that which has every angle acute.

Fig. 5. A trapezium is a quadrangle, whose opposite sides are not parallel.

All right-lined figures, having more than four sides, are called polygons, and receive their names from the number of their sides or angles.

Fig. 6. Having five sides or angles, is called a pentagon.

A regular polygon is a figure with equal sides and equal angles.

Fig. 7. A circle is a plane figure bounded by a curve line called the circumference, every part whereof is equally distant from a point within, called the centre.

A diameter, A B, of a circle, is a right line drawn through the centre, and terminated by the circumference.

The semi-diameter, A C, is called the radius.

A semi-circle is a figure contained under a diameter, and that part of the circumference of a circle cut off by that diameter, as the line A B divides the circle into two semi-circles.

Fig. 8. A segment is any part of a circle terminated by an arc, A D B, cut off by the line A B, called the chord.

Fig. 9. A sector of a circle is a part contained between two right lines or semi-diameters, and the intercepted arc of the circumference.

Fig. 10. Represents the front of an arch built with stones of equal length, and is a segment of a sector.

The hollow side, A B, of a curve, is called concave, and the raised side, C D, convex.

Fig. 11. An ellipsis, or oval, is a figure bounded by a regular curve line, returning into itself, but its two axes cutting each other in the centre; one of which is longer (called the transverse axis) than the other (called the conjugate axis).

A solid is that which hath length, breadth, and thickness.

Fig. 12. A cube is a solid bounded by six equal squares.

Fig. 13. A prism is a solid whose sides are parallelograms, and whose two ends are parallel to each other.

Fig. 14. A cylinder is a round solid, like the rolling-stone of a bowling-green, whose two ends are equal and parallel circles.

Fig. 15. A pyramid is a solid, whose base is a polygon, or right-lined figure, and whose sides, or triangles, meet in a point, C, called the vertex.

Fig. 16. A cone is a round pyramid, or pyramid having a circular base, in form like a sugar-loaf.

Fig. 17. 18. A frustum of a pyramid or cone is that part which remains, when any part next the vertex is cut off by a plane parallel to the base.

Fig. 19. A wedge is a solid, having a rectangular base, D B, and two of the opposite sides ending in an acies or edge, E F.

Fig. 20. A pavilion is a solid contained under five planes; the base is a rectangle or oblong, and the four sides terminate in a ridge, E F, parallel to a side of the base, A B, or C D, but unequal to it.

Fig. 21. A prismoid is a solid contained under six planes; the bases, or ends, are parallel rectangles, and the four sides are quadrangles.

Fig. 22. A sphere is a solid bounded by a convex surface, every point of which is equally distant from a point C, within, called the centre.

The axis, or diameter of a sphere, is the right line A B.

Fig. 23. A segment of a sphere is a part cut off by a plane A B. If the plane pass through the centre of the sphere, it will cut it equally in two, and each half is called a hemisphere.

Fig. 24. A spheroid is a solid resembling an egg, and is the body conceived to be generated by the revolution of an ellipse about its axis, and is denominated either prolate (oblong) or oblate, according as the revolution is made about the transverse axis or its conjugate.

The axis about which the revolution is made is the fixed axis, the other is the revolving axis.

Fig. 25. A parabolic spindle is eight-fifteenths of its circumscribing cylinder.

Fig. 26. Is the middle frustum of a spheroid.

LXXV. SUPERFICIAL MEASURE.

PROBLEM I.

To multiply feet, inches, and parts, by feet, inches, and parts, which method is termed cross multiplication, but more properly duodecimals.

RULE.

Set the feet in the multiplier under the least denomination in the multiplicand, and the rest in order, beginning with the least denomination; divide each product by 12, as you go on; place the first remainder under the multiplying figure, and the rest in order, adding each quotient to the next arising product,

as in Sect. IX. and having finished the multiplication, the sum of all will be the product required.

In general, thus;

When feet are concerned, the product is of the same denomination with the term multiplying feet.

When feet are not concerned, the name of the product will be expressed by the sum of the indices of the two factors.

EXAMPLES.

(1) Multiply 17 feet, 7 inches, by 6 feet.
(2) Multiply 47 feet, 8 inches, by 8 feet, 4 inches.
(3) Multiply 7 feet, 10 inches, by 8 feet, 6 inches.
(4) Multiply 64 feet, 7 inches, by 4 feet, 8 inches.
(5) Multiply 12 feet, 8 inches, 9 parts, by 9 feet, 6 inches, 7 parts.
(6) Multiply 9 feet, 11 inches, 6 parts, by 11 feet, 8 inches.
(7) Multiply 64 feet, 10 parts, by 14 feet, 9 inches.
(8) Multiply 124 feet, 4 inches, by 42 feet, 9 seconds.
(9) Multiply 16 feet, 7 inches, 10 parts, by 6 feet, 5 inches, 7 seconds.
(10) Multiply 474 feet, 6 inches, 8 seconds, by 186 feet, 7 inches, 4 seconds.
(11) Multiply 24 feet, 11′, 8″, 6‴, 7⁗, by 8 feet, 6′, 7″.
(12) Multiply 46 feet, 6 in. 8′, 4‴, by 6 feet, 4 in. 8′, 6‴.

PROBLEM II.

To find the area of a parallelogram, whether it be a square, a rectangle, a rhombus, or a rhomboides.

RULE.

Multiply the length by the height or perpendicular breadth, and the product will be the area.

If the area of a piece of ground in yards, be divided by 4840 (the number of square yards in one acre) the quotient will give the number of acres in the said piece. Or,

If the area in links be divided by 100,000 (the number of square links in one acre), the quotient will give acres.

That is AB × AC = the area.

Fig. 1.

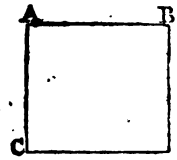


Fig. 2.

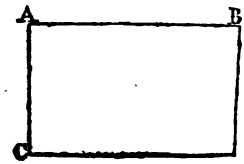
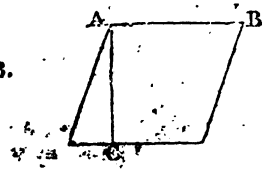


Fig. 3.



EXAMPLES.

(1) What is the area in acres of a parallelogram whose length is 14,5 chains, and its breadth 9,75 chains?

(2) What is the area of a square whose side is 245 yards?

(3) How many square yards of paving are there in a courtyard, being in the form of a rhombus or rhomboides, whose length is 64 feet, 6 inches, and perpendicular breadth is 47 feet, 8 inches?

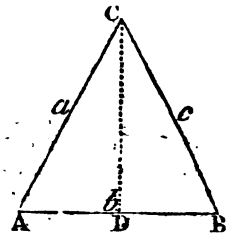
PROBLEM III.

Case 1. To find the area of a triangle.

RULE.

1. Multiply one of its sides by the perpendicular let fall upon it from its opposite angle, and half the product will be the area.
2. Multiply the base by half the perpendicular, or perpendicular by half the base, and the product gives the area.

Fig. 4.



That is, $\frac{AB \times CD}{2}$ = the area.

EXAMPLES.

(4) How many acres are in a triangular field, whose base is 28, and perpendicular 20,5 chains?

(5) A triangular field 738 links long, and 583 in the perpendicular, brings in 12*l.* a-year. What is it let at per acre?

Case 2. When the three sides of a triangle are given, to find the area.

RULE.

3. From half the sum of the three sides subtract each side severally; multiply the half sum and the three remainders continually together, and the square root of the last product will be the area of the triangle, that is, $\frac{a+b+c}{2} = s =$ half the sum of the sides.

Then let $s-a=e$, and $s-b=f$, also $s-c=g$ $\therefore \sqrt{sefg}$ = the area.——Note, $a=AC$, $b=AB$, and $c=BC$. See the last figure.

EXAMPLES.

(6) Suppose I have a fish-pond of a triangular form, whose three sides measure 400, 348, and 312 yards; what quantity of ground does it contain?

PROBLEM IV.

To find the area of a trapezium.

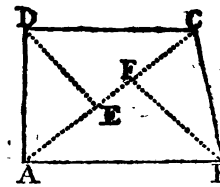
RULE.

1. Divide it into triangles, according to the manner which you judge most convenient; then the sum of the area of those triangles, calculated by the last problem, will be the area of the trapezium; Or,

2. Multiply the sum of the perpendiculars by half the diagonal, and the product will give the area; or multiply the sum of the perpendiculars by the diagonal, and half the product will be the area.

That is, $\frac{DE+BF}{2} \times AC =$ the area.

Fig. 5.



Or, 3. Subtract severally each side from half the sum of the four sides, and the square root of the product of the four remainders will be the area required.

EXAMPLES.

(7) How many square yards of paving are there in a trapezium, ABCD, whose diagonal, BD, 45 feet, and the perpendiculars, AE, equal to 17,25, CF, equal to 14 feet?

(8) Suppose the four sides of a trapezium are 15,60; 13,20; 10, and 26 chains: query, the area?

PROBLEM V.

To find the area of any regular polygon.

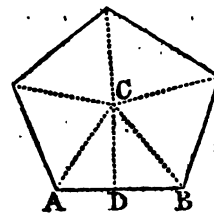
RULE I.

Let fall a perpendicular from the centre of the figure to one of its sides; then multiply together the perpendicular, the side of the figure, and the number of its sides, and half the product will be the area.

Here the number of sides is $5 = N$.

Then it is, $\frac{AB \times CD \times n}{2} =$ the area.

Fig 6.



EXAMPLE.

(9) A piece of garden-box lies in the form of a regular pentagon, or figure of five equal sides (as above), each 48 feet: and from the centre of the figure, C, to the middle of one of its sides, D, it measures 41,57 feet nearly: the area of the figure will be the contents of these five triangles. Pray what is that?

RULE.

Multiply the square of the side of any regular figure by the multiplier standing opposite to its name in the following table; and the product will be the area.

No. of Sides.	Names.	Multipliers.
3	Trigon or equal △	0,433013
4	Tetragon or Square	1,000000
5	Pentagon	1,720477
6	Hexagon	2,598076
7	Heptagon	3,633912
8	Octagon	4,828427
9	Nonagon	6,181824
10	Decagon	7,694209
11	Undecagon	9,365641
12	Dodecagon	11,196152

EXAMPLES.

(10) What is the area of a hexagon, whose side is 30?
(11) What is the area of an octagon, whose side is 24?

PROBLEM VI.

To find the diameter and circumference of a circle, the one from the other.

RULES.

1. Multiply the diameter by 3,1416, and the product will be the circumference. And therefore,
2. Divide the circumference by 3,1416, and the quotient will be the diameter.
3. See Sect. LIII. Case 3.

EXAMPLES.

(12) If the diameter of a circle be 7, what is the circumference?

(13) What is the diameter of a circle whose circumference is 22?

(14) What is the circumference of the earth, supposing it be perfectly round, and its diameter be 8000 miles?

PROBLEM VII.

To find the area of a circle.

RULES.

1. Multiply half the circumference by half the diameter, and the product will be the area. Or,
2. Multiply the square of the diameter by ,7854, and the product will be the area. Or,
3. Multiply the square of the circumference by ,079574, and the product will be the area. Or,
4. Multiply the square of the semi-diameter by 3,1416, and the product will be the area. Or,
5. Multiply the circumference by the diameter, and a fourth part of the product will express the area.

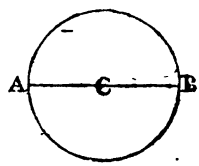
*** ,7854, and 3,1416, are areas of circles whose diameters are 1 and 2; and ,079577 is the area of a circle whose circumference is 1; likewise 452, and 1,273239, are squares of the diameters of circles, whose areas are 355 and 1; and 1,12837 is the diameter of a circle, whose area is equal to a square,

whose side is 1. For if the diameter be unity, or 1, the circumference will be 3,1415926, then per rule I. $\frac{3,1415926}{2} \times \frac{1}{2} = ,78539816$, or rather, ,7854, the area. Also, if the diameter be 2, the circumference will be 6,2831853; then per rule I. $\frac{6,2831853}{2} \times \frac{2}{2} =$ 3,1415926, or rather, 3,1416, the area. Likewise $\frac{1}{3,1416} \times 1 \div 4 = ,079477$, the area, &c.

Let C = the circumference, and D = the diameter, AB.

Then per rule I. $\frac{C}{2} \times \frac{D}{2} =$ the area.

Fig. 7.



EXAMPLES.

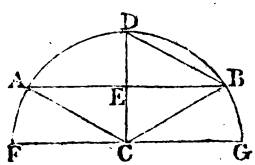
(15) How many square feet are in a circle whose circumference is 6,2832?

(16) What is the area of a circle whose diameter is 12?

PROBLEM VIII.

To find the length of any arc of a circle, ABD.

Fig. 8.



RULE.

Multiply together the radius, DC, the number of degrees in the given arc, and the number ,01745329 (b), the

last product will be the length of the arc; for, when the radius is 1, half the circumference is 3,14159265, &c. and therefore, $\frac{3,14159265}{180 \text{ degrees}} = ,01745329$, or ,0174533, nearly which is the length of an arc of 1 degree.

Hence CD × ADB × b = the length of the arc, ADB.

EXAMPLE.

(17) What is the length of the arc, ABD, which is 29,5 degrees, and radius 9?

PROBLEM IX.

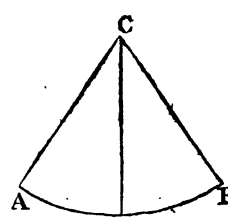
To find the area of any sector of a circle.

RULE.

Multiply the radius by half the arc of the sector, found by the last problem, and the product will be the area, as in the whole circle.

That is, $AC \times \frac{AB}{2}$ = the area.

Fig 9.



EXAMPLE.

(18) What is the area of a sector, whose radius, CA, is 55, and the length of the arc, AB, 59?

PROBLEM X.

To find the area of the segment of a circle, ADB, whose centre is E. (See Fig. 3.)

RULE.

Find the area of the triangle ABC, by Prob. III. and of the sector, ADBC, by the last problem: their difference, or sum of these areas, will be that of the segment, according as it is less or greater than a semi-circle. Or,

To six times the base AB (see Fig. 8.), add eight times the chord of half the arch AB, viz. DB; multiply the sum by the altitude DE, divide the product by 15, and the quotient will nearly give the area.

A TABLE of the segments of circles, whose area is unity or 1, the diameter being divided by parallel chord-lines into 100 equal parts.

V. S.	Segment.	V. S.	Segment.	V. S.	Segment.	V. S.	Segment.
1	,0017	26	,2066	51	,5127	76	,8155
2	,0048	27	,2178	52	,5255	77	,8262
3	,0087	28	,2292	53	,5382	78	,8369
4	,0134	29	,2407	54	,5509	79	,8474
5	,0187	30	,2523	55	,5635	80	,8576
6	,0245	31	,2640	56	,5762	81	,8677
7	,0308	32	,2759	57	,5888	82	,8776
8	,0375	33	,2878	58	,6014	83	,8873
9	,0446	34	,2998	59	,614	84	,8968
10	,052	35	,3119	60	,6265	85	,9059
11	,0598	36	,3241	61	,6389	86	,9149
12	,068	37	,3364	62	,6514	87	,9236
13	,0764	38	,3486	63	,6636	88	,932
14	,0851	39	,3611	64	,6759	89	,9402
15	,0941	40	,3735	65	,6681	90	,948
16	,1032	41	,3860	66	,7002	91	,9554
17	,1127	42	,3986	67	,7122	92	,9625
18	,1224	43	,4112	68	,7241	93	,9692
19	,1323	44	,4238	69	,756	94	,9755
20	,1424	45	,4365	70	,7477	95	,9813
21	,1526	46	,4491	71	,7593	96	,9866
22	,1631	47	,4618	72	,7708	97	,9923
23	,1738	48	,4715	73	,7822	98	,9952
24	,1845	49	,4873	74	,7934	99	,9983
25	,1955	50	,5	75	,8045	100	1,0000

EXAMPLES.

(19) Suppose the diameter, FG, of a circle to be 84 inches, and the height of the segment, ED, 30 inches, what will its area be?

(20) What is the area of a segment whose arc is a quadrant, or contains 90 degrees, and diameter 18 feet?

PROBLEM XI.

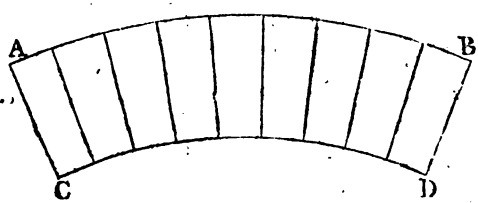
To find the area of a segment of a sector, ABCD, or the front of an arch built with stones of equal length.

RULE.

Multiply half the sum of the bounding arches, AB and CD, by the distance, AC, and the product will give the area.

That is, $\frac{AB+CD}{2} \times AC =$ the area nearly.

Fig. 10.



EXAMPLES.

(21) What is the area of the front of an arch built with stones $3\frac{1}{2}$ feet long, whose upper and lower bounding arches are in length 84 and $72\frac{1}{2}$ respectively?

(22) What is the area contained between two concentric semicircles, whose diameters are 24 and 16?

PROBLEM XII.

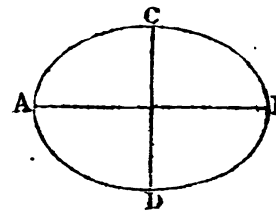
To find the area of an ellipsis, or oval.

RULE.

Multiply continually together the two axes, and the number ,7854 (b), and the product of these three numbers will express the area.

That is, $b \times AB \times CD =$ the area.

Fig. 11.



EXAMPLE.

(23) What is the area of an ellipsis whose greatest diameter is 24, and the least diameter 18?

OF ARTIFICERS' WORK.

I. Glaziers' and Masons' Flat Work is measured by the Foot Square.

EXAMPLES.

(1) What is the content of 12 panes of glass, each measuring 3 feet 10 inches long, and 2 feet 8 inches broad? What will the glazing come to at $8\frac{1}{2}d.$ per foot?

(2) There is a house with 3 tier of windows, 4 in a tier; the height of the first tier is 6 feet 6 inches, the second $5\frac{3}{4}$ feet, and the third $4\frac{1}{2}$ feet; the breadth of each window is 3 feet 9 inches. What will the glazing come to at $16d.$ per foot?

(3) What is the price of a marble slab, whose length is $6\frac{1}{8}$ feet, and the breadth $3\frac{3}{4}$ feet, at $8s.$ per foot?

(4) A looking-glass is 16 inches by 9, and contains a foot of glass. What will the content of the plate be, that has twice the length, and three times the breadth?

II. Painting, Plastering, Paving, &c. is measured by the yard square, which is 9 square feet.

RULE.

Divide the square feet by 9; and the quotient will be the number of square yards.

EXAMPLES.

(5) What will the paving of a street come to at 6*d.* per yard, the length of the street being 176$\frac{1}{2}$ feet, and the breadth 56$\frac{3}{4}$ feet?

(6) What is the content of a piece of wainscotting in square yards, that is 9$\frac{1}{2}$ feet in height, and 8$\frac{3}{4}$ feet broad? and what will it come to at 6*s.* per yard?

(7) There is a room 84 feet round, and 9 feet 6 inches high, in which are three windows, each 6 feet high, and 3 feet 5 inches wide, and the fire-place 4 feet by 4 feet. I demand how many yards of paper, half-yard wide, will hang it?

(8) If my court-yard be 47 feet 7 inches square, and I have laid a footway of Purbeck stone 4 feet wide, along one side of it, what will paving the rest with flints come to at 6*d.* per yard square?

(9) A rectangular four-sided room measures 129$\frac{1}{2}$ feet about, and is to be wainscotted at 3*s.* 6*d.* per yard square; after the due allowances for girt of cornice and member, it is 16$\frac{3}{4}$ feet high; the door is seven feet by 3$\frac{3}{4}$ feet; the window-shutters, two pair, are 7$\frac{1}{4}$ feet by 4$\frac{1}{2}$ feet; the check-boards round them come 1$\frac{1}{4}$ foot below the shutters, and are 14 inches in breadth; the lining-boards round the door-way are 16 inches broad; the door and window-shutters, being wrought on both sides, are reckoned work and half, and paid for accordingly; the chimney 3$\frac{3}{4}$ feet by 3 feet, not being enclosed, is to be deducted from the superficial content of the room; and the estimate of the charge is required.

(10) What will the plastering of a ceiling, at 10$\frac{1}{2}$*d.* per yard, come to, supposing the length 34$\frac{1}{2}$ feet, and the breadth 20 feet?

(11) There is a quantity of partitioning that measures 34 feet 8 inches about, and 14$\frac{1}{2}$ feet high; but is rendered between quarters: the lathing and plastering will be 8*d.* per yard, and the whiting 2*d.* per yard. What will the whole come to?

Note.—In measuring plastering; rendering between quarters, there is commonly a fifth part of the whole area deducted; but when rendering between quarters is whited or coloured, there is commonly a fourth or fifth

part added to the whole area, for the sides of quarters and braces, &c.

III. FLOORING, PARTITIONING, ROOFING, TILING, &c. is measured by the square of 100 feet.

IN these measurements the dimensions are taken by a rod of ten feet long; and therefore the result is in squares of 100 square feet each.

Hence dividing the area in square feet by 100, the quotient will be the number of squares required.

EXAMPLES.

(12) In $120\frac{1}{2}$ feet in length, and $12\frac{3}{4}$ feet in height of partitioning, how many squares?

(13) What difference is there between a floor 98 feet long, and 20 broad, and two others that measure 14 feet apiece by 10? and what do all three come to, at *2l. 5s.* per square?

(14) Suppose a house of three stories, besides the ground-floor, were to be floored at *8l. 10s.* per square; the house measures $30\frac{1}{2}$ feet by $20\frac{1}{2}$ feet; there are eight fire-places, whose measures are, four of 6 feet by $5\frac{3}{4}$, and four of $4\frac{3}{4}$ feet by 4, and the well-hole for the stairs is 10 feet by $8\frac{1}{2}$; what will the whole come to?

(15) How many oaken planks will floor a room $60\frac{1}{2}$ feet long, and $33\frac{1}{2}$ wide, supposing the plank 15 feet long, and $1\frac{3}{4}$ wide?

(16) Suppose a house measures, within the walls, 64 feet in length, and 36 feet in breadth, and to be of a true pitch; what will it come to roofing, at *12s. 6d.* the square?

(17) Suppose I employ a person to thatch a barn, which is 70 feet long, and 30 deep; I demand how many squares are contained in the whole; also what it will come to at *10s. 8d.* per square?

(18) What will the new ripping an out-house cost, that measures $32\frac{3}{4}$ feet long, by $22\frac{3}{4}$ broad, upon the flat, at *15s.* the square; the eaves' boards projecting 10 inches on each side?

Note.—In tiling and roofing, it is customary to reckon the flat and half of any building within the walls to be

the depth or width of the roof of that building when the said roof is of a true pitch, that is, when the rafters are $\frac{1}{2}$ of the breadth of the building. But when the roof is more or less than the true pitch, they measure from one side to the other.

IV. BRICKLAYERS' WORK is measured by the rod, of $272\frac{1}{4}$ square feet.

THIS work is always valued at the rate of a brick and a half thick; and if the thickness of the wall be more or less, it must be reduced to that thickness by the following

RULES.

1. Multiply the area of the wall in feet, by the number of half bricks in the thickness the wall is of: divide the product by $816\frac{1}{4}$, and the quotient will be the content in rods; or,
2. Multiply the area of the wall by the number of half bricks the thickness the wall is of; the product, divided by 3, gives the area in feet, which divide by $272\frac{1}{4}$; the quotient will be the rods required.

Note.—The fraction $\frac{1}{4}$ in rule 1, or $\frac{1}{4}$ in rule 2, is rejected in favour of the workmen.

EXAMPLES.

(19) There is a brick wall 470 feet round, and $9\frac{1}{2}$ feet high, and three bricks thick. How many rods does it contain?

(20) A gentleman built a wall round his garden, which is 840 feet, and 9 feet high, and $2\frac{1}{2}$ bricks thick. How many rods does it contain, and what will it come to at 4*l.* 19*s.* 6*d.* per rod?

(21) The end wall of a house is $24\frac{1}{2}$ feet in breadth, and 40 feet to the roof; $\frac{1}{2}$ of which is two bricks thick, $\frac{1}{4}$ more $1\frac{1}{2}$ brick thick, and the rest 1 brick thick. Now the gable rises 38 course of bricks (4 of which usually make a foot in depth), and this is but 4 inches, or half a brick thick. What will this piece of work come to at 5*l.* 10*s.* per statute rod?

QUESTIONS *for Exercise in* SUPERFICIAL MEASURE.

(1) An elm plank is 14¼ feet long, and I would have just a yard square slit off. At what distance from the edge must the line be struck?

(2) Having a rectangular marble slab, 58 inches by 27, I would have a foot square cut off, parallel to the shorter edge. I would then have the like quantity divided from the remainder, parallel to the longer side: and this alternately repeated till there be not the quantity of a foot left. What will the dimension of the remainder be?

(3) Being about to plant 10584 trees equally distant in rows, the length of the grove must be six times the breadth. How many of the shorter rows will there be?

(4) A common joist is 7 inches deep, and 2¼ thick. But I want a scantling just as big again, and that shall be three inches thick. What will the other dimensions be?

(5) I have a square girder, 19 inches by 11; but one quarter of the timber in it (provided it be 9 inches deep) will serve. How broad will it be?

(6) I have a wooden trough, that, at 6*d.* per yard, cost me 3*s.* 2*d.* painting within; the length of it was 102 inches, the depth 21 inches; what was its breadth?

(7) My plumber has put 28 lb. per foot square into a cistern 74 inches, and twice the thickness of the lead long, 26 inches broad, and 40 deep; he has put three stays within, across it, 16 inches deep, of the same strength; and reckons 22*s.* per cwt. for work and materials: I being a mason, have paved him a workshop, 22 feet 10 inches broad, with Purbeck stone, at 7*d.* per foot, and upon the balance I find there is 3*s.* 6*d.* due to him. What was the length of his workshop?

(8) The rectangular powdering trough of a man of war measures 27 square feet, 112 inches; the depth is 20 inches, the breadth 16. The length is sought.

(9) In 110 acres of statute measure, in which the pole is 16½ feet long, how many Cheshire acres, where the customary pole is 6 yards long; and how many Yorkshire, where the pole in use is 7 yards in length?

(10) I would set 3584 plants in rows, each 4 feet asunder, and the plants 7 feet apart, in a rectangular plot of ground. What land will this take up?

(11) The paving of a triangular court, at 18*d.* per foot, came to 100*l.* the longest of the three sides was 88 feet; what then was the sum of the other two equal sides?

(12) An ancient bath was found of a triangular form, the sum of whose three equal sides was 125 feet: the area of the bottom is required?

(13) I would plant 10 acres of hop ground, which must be done either in the square order, as the number 4 stands on the dice, or in the quincunx order, as the number 5; the three nearest blinds, in both cases, must be set linealy just 6 feet asunder. How many plants more will be required for the last order than for the first, admitting the form of the plot to lie the most advantageous for the plantation in either case?

(14) A summer-house is a cube of 10 feet in the clear; the cornice projects just 15 inches on a side; and being of timber and stucco, the sides are 6 inches thick, so that the whole front of the roof, from out to out, is $13\frac{1}{2}$ feet; this is hipped from each of the corners to the centre, and being truly pediment pitch, it rises $\frac{2}{6}$ of the front, or 3 feet. I would, by help of these dimensions, measure the slating without venturing to climb for more, and compute the cost $3\frac{1}{2}$*d.* per square foot.

(15) A triangular bath, 6 feet deep, is exactly enclosed by 3 square pavilions, and rectangular, the sum of whose planes together make just 50 poles. The area of A. the less is to that of B. the middle one, as $4\frac{1}{2}$ to 8; and the sum of the areas of A. and C. the biggest, is to that of B. as $8\frac{1}{2}$ to 4. How many wine hogsheads of water will this bath receive?

(16) I have an orchard in the form of a quadrangle or trapezium, containing $3\frac{1}{4}$ acres, which being divided by a diagonal, or line from corner to corner, the perpendicular of one of the triangles is 430 links, and the other 360. The length of the said diagonal, or common base of those triangles, is required.

(17) Give the area of a circular bowling green, that is 16 poles across the middle, the circumference being 3,1416 times the diameter of a circle.

(18) The surveying wheel is so contrived as to turn just

twice in the length of a pole, or 16¾ feet. What then is the diameter?

(19) I would turf a round plot, measuring 130 feet about, and would know the charges, at 4*d.* per yard square.

(20) I want the length of a line, by which my gardener may strike a round aurangerie, that shall contain just half an acre of land.

(21) Agreed for an oaken curb to a round well at 8*d.* per square foot; it is exactly 42 inches in diameter, within the brick work, and the breadth of the curb is to be 14½ inches. What will it come to?

(22) It is observed, that the extreme end of the minute-hand of a public dial moves just 5 inches in the space of 5¼ minutes. The question is, what is the length of that index?

(23) A. B. and C. join for a grinding stone, 36 inches in diameter, value 20*s.* towards which A. paid 7*s.* B. 8*s.* and C. 5*s.* the waste hole through which the spindle passed was 5 inches square. To what diameter ought the stone to be worn, when B. and C. begin severally to work with it?

(24) I demand what difference there is in the area of the section of a round tree, 20 inches over, and its inscribed and circumscribed squares.

(25) Having paved a semicircular alcove with black and white marble, at 2*s.* 4*d.* per foot, the mason's bill was just 10*l.* What then was the arch in front?

(26) What proportion is there between the arpent of France, which contains 100 square poles of 18 feet each, and the English acre, containing 160 square poles of 16½ feet each, considering that the length of the French foot is to the English as 16 to 15?

(27) In turning a one-horse chaise within a ring of a certain diameter, it was observed that the outer wheel made two turns, while the inner made but one. The wheels were equally high; and supposing them fixed at the statutable distance, or 5 feet asunder on the axle-tree, pray what was the circumference of the track described by the outer wheel?

(28) Required the area of a sector (supposing one of the divisions of a wilderness), which, being struck from a centre with a line 30 yards long, makes the sweep or circular part 63 feet?

(29) If the chord or line drawn through the two ends of the above curve be 15 inches shorter than the arch-line, I demand the segment.

(30) Suppose the ellipse in Grosvenor-square measure 840 links the longest way, and 612 across, within the rails, and if the curb stones be 14 inches thick, what ground do they inclose, and what do they stand upon?

Note.—The dimensions of all similar figures are in proportion to their areas, as the squares of their respective sides, *et contra.*

(31) If a round pillar, 7 inches over, have 4 feet of stone in it, of what diameter is the column, of equal length, that measures ten times as much?

(32) A pipe of 6 inches bore will be 3 hours in running off a certain quantity of water. In what time will 4 pipes, each 3 inches bore, be in discharging double the quantity?

(33) Suppose a yard of rope, 9 inches round, weigh 22 lb. what will a fathom of that weigh which measures a foot about?

(34) If 20 feet of iron railing shall weigh half a ton when the bars are an inch and quarter square, what will 50 feet of ditto come to, at $3\frac{1}{2}d.$ per lb. the bars being but $\frac{7}{8}$ of an inch square?

(35) A sack that holds three bushels of corn is $22\frac{1}{2}$ inches broad when empty; what would the sack contain, that, being of the same length, had twice its circumference, or twice its breadth?

(36) My plumber has set me up a cistern; and his shop-book being burnt, he has no means of bringing in the charge; and I do not choose to take it down to have it weighed; but by measure he finds it contains 64 square feet $\frac{3}{10}$, and that it was $\frac{3}{8}$ of an inch precisely in thickness. Lead was then wrought at 21*l.* per fother. Let the accomptant, from these items, make out the poor man's bill, considering further, that $4\frac{4}{11}$ oz. is the weight of a cubic inch of lead.

LXXVI. MENSURATION of SOLIDS.

PROBLEM XIII.

To find the solidity of a cube, prism, or right cylinder.

Fig. 12.

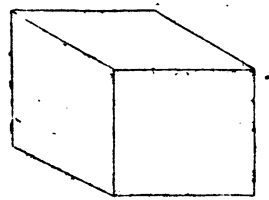


Fig. 13.

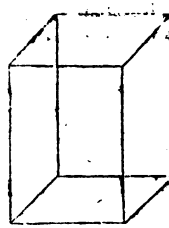
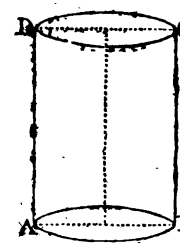


Fig. 14.



RULE.

Multiply the area of the base into the height or altitude, and the product will be the solidity.

EXAMPLES.

(1) What is the solid content of a cube, whose side is $2\frac{1}{2}$ feet?

(2) How many ale gallons of water will a cistern hold, whose length, breadth, and depth, are 4 feet 9 inches, 3 feet 6 inches, and 2 feet 10 inches?

(3) What is the content of a cylinder, whose diameter is $4\frac{1}{2}$ feet, and 8 feet high?

PROBLEM XIV.

To find the convex surface of a prism, or a right cylinder.

A GENERAL RULE.

Find the area of each side and end separately, then add those areas together, and their sum will be the whole surface of any prism or body whatever.

A PARTICULAR RULE.

Multiply the circumference of the base by the altitude of one cylinder, and the product will give the convex surface.

EXAMPLE.

(4) What is the convex surface of a right cylinder, whose circumference is $10\frac{1}{2}$ feet, and height $7\frac{3}{4}$ feet?

PROBLEM XV.

To find the solidity of a pyramid, or right cone.

Fig. 15.

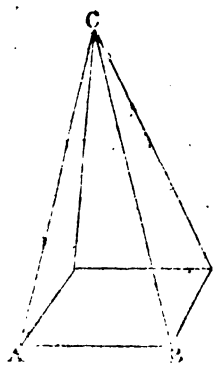
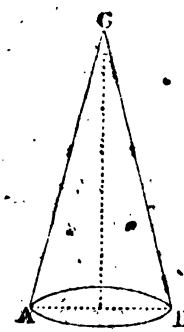


Fig. 16.



RULE.

Multiply the area of the base by a third part of the altitude, and the product will be the content required.

EXAMPLES.

(5) Required the solidity of a square pyramid, each side of whose base is 12 feet, and the slant height 25 feet.

(6) What is the solid content of a triangular pyramid, whose height is 30 feet, and each side of its base $5\frac{1}{2}$?

(7) What is the solidity of a cone, whose base is $3\frac{1}{2}$ feet diameter, and altitude 6 feet?

PROBLEM XVI.

To find the convex surface of a pyramid or cone (as Fig. 15 and 16).

RULE.

Multiply the perimeter or circumference of the base by the slant height or length of the side (AC) and half the product will be the area.

EXAMPLES.

(8) What is the surface of a triangular pyramid, including the base, the slant height being 20 feet, and each side of the base $3\frac{1}{4}$?

(9) What is the convex surface of a right cone, whose base is 45 feet in circumference, and slant side is 20 feet in length?

PROBLEM XVII.

To find the solidity of a frustum of a pyramid or cone.

Fig. 17.

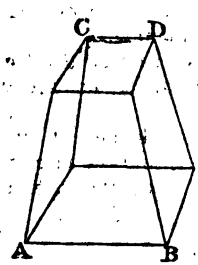
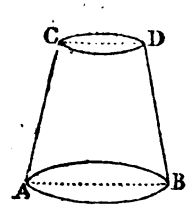


Fig. 18.



Add into one sum the areas of the two ends, and the mean proportional between them; multiply the sum by the

perpendicular height; and $\frac{1}{3}$ of the product will be the solidity: that is, if A be the area of the greater end, and a of the lesser, and h the height.

Then $A + a + \sqrt{Aa} \times \frac{1}{3}h =$ the solidity.

EXAMPLES.

(10) How many solid feet are there in a tree, whose bases are squares, each side of the one being 15 inches, and each side of the other 6, and the length measures along the side 24 feet?

(11) What is the content of a frustum of a cone 60 feet high, the diameters of its ends being 20 and 3 feet?

(12) How many solid feet are there in a conical frustum, the circumferences of whose bases are 66 and 56 feet, height is 4 feet?

PROBLEM XVIII.

To find the convex surface of the frustum of a pyramid or right cone.

RULE.

Multiply the sum of the perimeters or circumferences of the ends by the slant height, and half the product will be the surface required.

EXAMPLES.

(13) How many square feet are in the surface of a frustum of a square pyramid, whose slant height is 10 feet, each side of the greater basis being 3 feet 4 inches, and each side of the less 2 feet 2 inches?

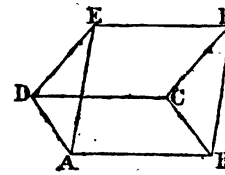
(14) How many square feet are in the surface of a frustum of a cone, whose circumferences of its ends are 32 and 8 feet, and slant side 7 feet?

(15) If a segment of 6 feet slant height be cut off a cone, whose slant height is 30 feet, and the circumference of its base 10 feet, what will be the surface of the frustum?

PROBLEM XIX.

To find the solidity of a cuneus or wedge.

Fig. 19.



RULE.

Multiply the area of the base, ADE, or BCF, by half the altitude, DC, of the wedge, and the product will give the solidity.

Thus, $AD \times DE \times \frac{CD}{2} =$ the solidity.

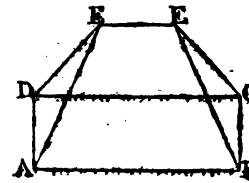
EXAMPLE.

(16) What is the solidity of a wedge, whose base measures 30 feet by 16, and whose height is 12?

PROBLEM XX.

To find the solidity of a pavilion roof.

Fig. 20.



RULE.

To the length of the ridge, add twice the side of the base which is parallel to it: multiply the sum by the other side of the base, and the product which arises by a sixth part of the altitude; and the second product will give the solidity.

Thus, $\overline{EF + 2AB} \times BC \times \frac{alt}{6}$.

(17) What is the solidity of a pavilion roof, whose base is 36 by 20, ridge parallel to the greatest side 16, and altitude 12 feet?

PROBLEM XXI.

To find the solidity of the frustum of a square pyramid made by a section parallel to the base.

RULE.

To the area of the ends add the product of their sides; multiply the sum by a third part of the altitude; and the product will give the solidity.

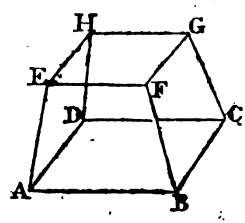
EXAMPLE.

(18) What is the solidity of the frustum of a pyramid 60 feet high, whose ends are 16 and 13 feet square?

PROBLEM XXII.

To find the solidity of a prismoid.

Fig. 21.



RULE.

To the area of the ends add the product of the sums of the lengths and breadths; multiply this sum by a sixth part of the altitude; and the product will give the solidity.

Thus, $AB \times BC + EF \times FG + \overline{AB + EF} \times \overline{BC + FG} \times \frac{alt.}{6}$.

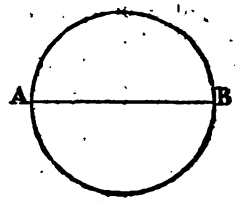
EXAMPLE.

(19) What is the solid content of a canal 304 feet by 20 at top, 300 feet by 16 at bottom, and 5 feet deep?

PROBLEM XXIII.

To find the surface of a sphere or globe, or of any segment or zone of it.

Fig. 22.



A GENERAL RULE.

Multiply the circumference of the sphere into the diameter or height of the part required; and the product will be the curve surface, whether it be segment, zone, hemisphere, or the whole sphere.

Note.—The height of the whole sphere is its diameter.

PARTICULAR RULES.

1. Find the circumference of a great circle upon the globe, by Prob. VI. Rule I. or by multiplying the radius by 6,2832: multiply the circumference by the diameter; the product will give the superficies.—Or,

2. Multiply 3,1416 by the square of the diameter, and the product will give the superficies.

EXAMPLES.

(20) What is the surface of a globe, whose diameter is 7?

(21) What is the surface of a globe, whose semi-diameter is 6 inches?

(22) If the diameter, or axis of the earth be 7957¾ miles, what is the whole surface, supposing it a perfect sphere?

(23) What is the superficies of a segment 9 feet high, cut from a globe of 42 feet diameter?

PROBLEM XXIV.

To find the solidity or content of a sphere or globe. (See Fig. to Prob. XXIII.)

RULE.

1. Find the superficies by the last Problem; multiply the superficies by $\frac{1}{3}$ of the radius, or by $\frac{1}{6}$ of the diameter; and the product will be the solidity—Or,

2. Multiply the cube of the diameter by ,5236; and the product will give the solidity; that is,

$$\frac{3,1416}{6} = ,5236, \text{ the solidity.—Or,}$$

3. Find the content of a circumscribing cylinder, by Problem XIII. and take $\frac{2}{3}$ of it for the content of the globe.

For a globe is $\frac{2}{3}$ of its circumscribing cylinder, and ,5236 is the content of a globe whose diameter is 1.

EXAMPLES.

(24) What is the content of a globe whose diameter is 7?

(25) Suppose the earth to be spherical, and its diameter $7957\frac{3}{4}$ miles, what is its solidity?

PROBLEM XXV.

To find the solidity of the segment of a globe. (See Fig. 23.)

RULE.

1. From three times the diameter of the globe, take twice the altitude of the segment; multiply together the remainder, the square of the altitude, and ,5236; and this product will give the solidity.

Thus let $h = CD$ the height of the segment, and d the diameter.

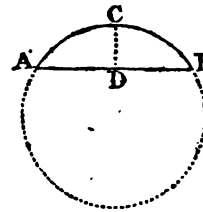
Then $3d - 2h \times ,5236 \times hh =$ the solidity of ACB.—Or,

2. To three times the square of the radius of its base, AB, add the square of its height; multiply the sum by the height, and that product again by ,5236, will give the solidity.

That is, if $r = AD$, the radius of its base, $h = CD$, the

height. Then $,5236h \times 3rr \times hh$=the solidity of the segment A B C.

Fig. 23.



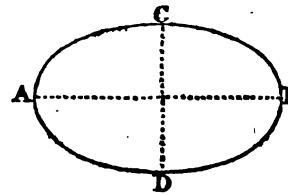
EXAMPLE.

(26) What is the solidity of a segment 4 feet high, cut from a globe 18 feet diameter?

PROBLEM XXVI.

To find the solid content of a spheroid.

Fig. 24.



RULE I.

Multiply continually together the fixed axis, the square of the revolving axis, and the number ,5236 (being $\frac{1}{6}$ of 3,14159 nearly; and the last product will be the content required; that is, if p=3,14159, &c. t=the transverse, and c=the conjugate axis of the generating ellipse;

Then $\frac{1}{6}$ ptc=the oblate.
And $\frac{1}{6}$ $ptcc$=the oblong spheroid.

RULE II.

Multiply the area of the generating ellipse by $\frac{2}{3}$ of the revolving axis, and the product will be the content of the spheroid.

Let A=the area of the ellipse: then from the former rule
$\frac{2}{3}tA$=the oblate,
And $\frac{2}{3}cA$=the oblong spheroid.

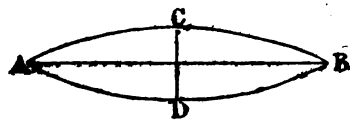
EXAMPLE.

(27) What is the solid content of a spheroid, whose diameter of the greatest circle is 33 inches, and the length 55 inches?

PROBLEM XXVII.

To find the solidity of a parabolic spindle.

Fig. 25.



RULES.

1. The square of the diameter (C D) of the greatest circle multiplied by ,41888 (being $\frac{3}{15}$ of ,7854), and that product again by its length (AB) will be the solidity.—Or

2. Multiply the area of the greatest circle, or middle section by the length; and $\frac{3}{15}$ of the product will be the content.

That is, if AB=the length or axis, DC=the greatest diameter, or double the abscissa of the generating parabola ACB, and n=,785398, or ,7854. Then $\frac{3}{15}n \times DC2 \times AB$= the whole solid ADBCA.

EXAMPLE.

(28) What is the solidity of a parabolic spindle, whose greatest diameter is 36, and its length 99 inches?

PROBLEM XXVIII.

To measure Timber.

A square piece of timber, equally thick at both ends, is a prism; a round piece, equally thick at both ends, is

a cylinder; a square piece, that tapers regularly, is the frustum of a pyramid; and a round piece, that tapers regularly, is the frustum of a cone; and the contents of these solids may be exactly computed by their respective rules.

But because the mensuration of tapering timber by the exact rules is troublesome, an approximation has taken place; and the contents of such trees are generally computed by the following

RULE.

Multiply the square of the quarter girt (or $\frac{1}{4}$ of the circumference) in inches, by the length in feet; divide the product by 144; and the quotient will be the content in feet.

But to find the content more near the truth, observe the following

RULE.

Multiply the square of $\frac{1}{5}$ of the girt or circumference by twice the length, and the product will be the content, extremely near the truth.

REMARKS.

1. The girt of a piece of timber is its compass or circumference at the middle, $\frac{1}{4}$ of which is commonly taken for the side of a square, equal to the area of the section there.
2. Trees of irregular growth must be measured in parts or pieces, as above directed.
3. Allowance must be made for the thickness of bark (if on the tree) in oak $\frac{1}{10}$ or $\frac{1}{12}$, in other wood not so much.
4. When the timber is to be reduced to loads,

$$\text{Divide the feet in } \left\{\begin{matrix}\text{rough,}\\ \text{hewn,}\end{matrix}\right\} \text{ by } \left\{\begin{matrix}40\\ 50\end{matrix}\right\} \text{ gives the loads;}$$

as 40 feet make one load of rough timber, and 50 one of hewn.

EXAMPLES.

(29) What is the content of a tree, whose girt is 42 inches, and length 16 feet?

(30) What is the content of a tree, whose compass is 64 inches, and the length $30\frac{1}{2}$ feet?

(31) How many loads of timber are there in a hewn tree, whose breadth is 42 inches, depth 30 inches, and length 40 feet?

GAUGING.

PROBLEM XXIX.

To find the area of any triangular tun, back, cooler, or circular and elliptical superficies, in ale gallons, &c.

RULES.

1. Find the area in inches by the different Problems in Sect. LXXV. and the solidity by Sect. LXXVI.; then,

$$\text{Divide by} \begin{cases} 282 \\ 231 \\ 268,8 \end{cases} \text{for} \begin{cases} \text{Ale,} \\ \text{Wine,} \\ \text{Corn,} \end{cases}$$

and the quotient will be the area in gallons.—Or,

2. If the square of the diameter of any circle;

$$\text{Divide by} \begin{cases} 359,05 \\ 294,12 \\ 342,24 \end{cases} \text{for} \begin{cases} \text{ale gallon,} \\ \text{wine gallon,} \\ \text{corn gallon,} \end{cases}$$

and the quotient will be the area in their respective gallons.

For as ,785398 : 1 :: 282 : the square of the diameter of the circle whose area is 282 cubic inches, viz. one ale gallon: and from this proportion arises the preceding divisions:

$$\text{Viz.} \begin{cases} 282 \\ 231 \\ 268,8 \end{cases} \div ,785398 = \begin{cases} 359,05 \\ 294,12 \\ 342,24 \end{cases}$$

Or, these divisors may be turned into multiplicators, by dividing unity or 1, or rather by dividing the area in inches, of that circle whose diameter is 1.

That is ,785398 by 282, &c.

$$\text{Thus, } ,785398 \div \begin{cases} 282 = ,002785 \\ 231 = ,00399 \\ 268,8 = 002922 \end{cases}$$

The product will be the area in gallons of the same name.

EXAMPLES.

(32) Suppose the length of a brewer's tun, back, or cooler, be 16 feet 6 inches, and its breadth 7 feet 4 inches, what will be the area in ale or beer gallons, &c.?

(33) The length of the base of a triangular cooler is 94 inches, and its perpendicular breadth is 58 feet 6 inches. Required its area in ale gallons.

(34) Suppose the longest diameter of a brewer's vessel be 84,5 inches, and the shortest diameter to be 50 inches; what will be the area in ale gallons?

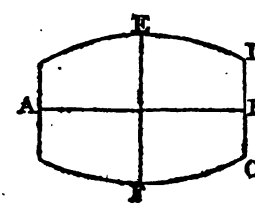
(35) Suppose a tun in the form of the frustum of a pyramid, whose bases are equilateral triangles: let the side of the top be 64 inches, the side of the bottom be 98,6 inches, and its height or depth 36 inches; what is the content of that tun in ale gallons, &c.?

(36) If the diameter of the base of a regular cone be 60,5 inches, and the perpendicular height be 42,8 inches, what will be the content in ale gallons, &c.?

(37) Suppose the diameter of a frustum of a cone be 84 inches at the top, and the diameter at the bottom be 62 inches, and the height 42 inches: required the content in ale gallons.

The bung diameter E F, head diameter C D, and length of the cask A B (withinside) being given; to find the content of a cylinder nearly equal to it.

Fig. 26.



RULE.

1. To twice the area of the circle at the bung, add the area of the circle of the head; multiply the sum by one-third of the length of the cask; the product is the content

in cubic inches, which are converted into gallons by dividing by 282 for ale, and 231 for wine gallons.—Or,

2. To the square of the head diameter, add twice that of the bung diameter, and from that sum take $\frac{2}{5}$ of the square of the difference of the said diameters: then multiply the remainder by the length of the cask: then if the product be multiplied

by $\left\{\begin{matrix} ,00092837 = 002787 \\ ,00113333 = 003399 \end{matrix}\right\} \times 3$, will give the area.

or divided by $\left\{\begin{matrix} 1077,15 = 359,05 \\ 882,36 = 294,12\frac{1}{3} \end{matrix}\right\} \times 3$, will give the area.

EXAMPLES.

(38) What is the content of a cask, whose bung diameter, head diameter, and length, are 32, 26, and 40 inches, withinside, respectively?

(39) Suppose the bung diameter of a cask to be 40 inches, head 36, and length 64: required the contents both in ale and wine gallons.

Questions for Exercise in Mensuration of Solids.

(1) What is the difference between a solid half foot, and half a foot solid?

(2) What is the proportion, in space, between a room 25 feet 6 inches long, 20 feet two inches broad, 14 feet high, and two others of just half the dimensions?

(3) Another room is 17 feet 7 inches long within, 13 feet 10 inches broad, and $9\frac{1}{2}$ feet high; it has a chimney carried up straight in the angle, the plan whereof is just the half of $5\frac{1}{2}$ feet, by 4 feet 2 inches. The question is, How many cubic feet of air the same will contain, allowing the content of the fire-place and windows at 4 solid yards?

(4) A ship's hold is $112\frac{1}{2}$ feet long, 32 broad, and $5\frac{1}{2}$ deep. How many bales of goods, 3 feet 4 inches long, 2 feet 2 inches broad, and 3 feet deep, may be stowed therein, leaving a gang-way $4\frac{1}{2}$ feet broad?

(5) I want a rectangular cistern, that, at 16 lb. to the foot square, shall weigh just a fother of lead; it must be

8 feet long, and $4\frac{1}{4}$ over; how many hogsheads, wine measure, will this cistern contain, taking it at $\frac{1}{4}$ of an inch from the top?

(6) A log of timber is $18\frac{1}{2}$ feet long, 18 inches broad, and 14 inches thick, die square all through. Now, if 2 solid feet and a half be sawed off the end, how long will the piece then be?

(7) The solid content of a square stone is found to be $126\frac{1}{4}$ feet, its length is $8\frac{1}{3}$ feet. What is the area of one end, and what the depth, if the breadth assigned be $38\frac{1}{2}$ inches.

(8) The dimensions of the circular Winchester bushel are $18\frac{1}{2}$ inches over, and 8 inches deep. How many quarters of grain will a square bin hold, that measures 7 feet 10 inches long, 3 feet 10 broad, and 4 feet 2 inches deep, within?

(9) Taking the dimensions of the bushel as above, what must the diameter of a circular measure be, which at 12 inches deep will hold 9 bushels of sea coal struck?

(10) A prism of two equal bases, and 6 equal sides, that measures 28 inches across the centre, from corner to corner; the superficial and the solid content is required, taking the length at 134 inches.

(11) I have a rolling stone 44 inches in circumference, and am to cut off three cubic feet from one end. Whereabouts must the section be made?

(12) I would have a syringe, $4\frac{1}{4}$ inch in the bore, to hold a pint (wine measure) of any fluid. What must the length of the piston, sufficient to make an injection with it, be?

(13) I would have a cubic bin made capable of receiving just $13\frac{1}{8}$ quarters of wheat, Winchester measure; what will be the length of one of its sides?

(14) A Bath stone, 20 inches long, 15 over, and 8 deep, weighs 220 lb. How many cubic feet thereof will freight a ship of 290 tons?

(15) The common way of measuring timber being to girt a round straight tree in the middle, and to take $\frac{1}{4}$ of the girt for the side of a square, equal to the area of the section there; if this be not considered in the price appointed, pray on which side lies the advantage?

(16) The solid content of a globe 20 inches in diameter, a cylinder of the same diameter, 20 inches long, and a cone 20 inches diameter at the base, and 20 inches

high, are severally required; and also what they will cost painting at 8*d*. per yard.

(17) Our satellite, the moon, is a globe, in diameter 2170 miles. I require how many quarters of wheat she would contain, if hollow, 2150$\frac{2}{3}$ solid inches being the bushel; and how much yard-wide stuff would make her a waistcoat, were she to be clothed.

(18) Suppose the atmosphere, or body of the air and vapours surrounding the globe of the earth and sea, to be 60 miles above the surface, and the earth is 7970 miles in diameter: how many cubic yards of air then hang about and revolve along with this planet?

(19) A square pyramid, whose sides at the base measure 10 inches a-piece, and is 20 feet high by the slope in the middle of each side of the base, is to be sold at 7*s*. per solid foot; and if the polishing the surface of the sides will be 8*d*. per foot more; I would know the cost of this stone when finished.

(20) A round mash-vat measures at the top 72 inches over within, at the bottom 54, the perpendicular depth being 42 inches; the content in ale gallons is required.

(21) The shaft of a round pillar, 16 inches in diameter at the top, is about 8 of the bottom diameters in height, $\frac{1}{3}$ whereof is truly cylindrical, and the other $\frac{2}{3}$ swelling; but we will suppose it tapers straight, and that it is $\frac{1}{6}$ less at the top than at bottom: the price of the stone and workmanship is sought, at 3*s*. 6*d*. per cubic foot; and farther, the superficial content, including both ends.

(22) A stick of square timber tapers straight; the side of the greater end is 19$\frac{1}{2}$ inches, of the less 13$\frac{1}{2}$ inches, the length 16 feet 6 inches; the value at 2*s*. 6*d*. per foot solid, is demanded.

(23) What quantity of brandy will the distiller's tun contain, that measures 40 inches within at the head, 52 at the bung, and 103 inches long; and how many barrels of London ale would fill it?

(24) Suppose the globe or ball, on the top of St. Paul's Church, to be 6 feet in diameter; what did the gilding thereof come to at 3$\frac{1}{2}$*d*. per inch square?

(25) The famous tun of Heidelberg, that being heretofore annually replenished with Rhenish, had in it some wine that was many ages old before the French de-

molished it in the late war. It was 31 feet in length, and 21 in diameter, and pretty nearly cylindrical. Pray how many tons of wine would the same contain?

LXXVII. SPECIFIC GRAVITY OF METALS.

THE specific gravity of a body is the relation that the weight of a body of one kind hath to the weight of an equal magnitude of a body of another kind; the knowledge of which is of great use in computing the weights of such bodies as are too unwieldy to have their weight discovered by other means.

The following TABLE shows the specific gravity, to rain-water, of metals, and other bodies; and the weight of a cubic inch of each, in parts of a pound avoirdupois, and in ounces troy, and parts of an ounce.

BODIES.	Sp. Grav.	wt. lb. Avois.	wt. oz. Troy.
Fine gold	19,640	0,7103587	10,439273
Standard gold	19,520	0,7060185	9,962625
Coast gold	18,868	0,6828703	9,911707
Quicksilver	13,762	0,4976574	7,384411
Lead	11,313	0,4091696	5,964010
Fine silver	11,091	0,4011501	5,850035
Standard silver	10,629	0,3810402	5,556769
Cast silver	10,528	0,3807870	5,503967
Copper	8,769	0,3171658	4,747121
Plate brass	8,350	0,2942593	4,404273
Cast brass	8,104	0,2929832	4,272409
Steel	7,850	0,2839265	4,142127
Bar iron	7,704	0,2808459	4,031361
Block tin	7,238	0,2417901	3,861519
Cast iron	7,135	0,2580647	3,806568
Loadstone	5,106	0,1846708	2,724183
Blue slate	3,500	0,1264914	1,867272
Veined marble	2,702	0,0977286	1,429411
Common glass	2,600	0,0940393	1,380841
Flint stone	2,582	0,0933883	1,331419
Portland stone	2,570	0,0929543	1,345139
Free stone	2,252	0,0815766	1,231038
Brick	2,060	0,0723379	1,046801

BODIES.	Sp. Grav.	wt. lb. Avoir.	wt. oz. Troy.
Alabaster	1,883	0,0683061	0,988450
Ivory, Horn	1,832	0,0662606	0,958489
Brimstone	1,800	0,0651042	0,949424
Clay	1,712	0,0619213	0,902495
Lignum vitæ	1,327	0,0479962	0,699986
Coal	1,255	0,0453921	0,661959
Pitch	1,150	0,0515943	0,606759
Mahogany wood	1,063	0,0384475	0,560691
Dry box wood	1,030	0,0372530	0,543282
Milk, Sea water	1,033	0,0372530	0,543742
Rain water	1,000	0,0361690	0,527458
Red wine	0,993	0,0359158	0,523766
Bees' wax	0,995	0,0359881	0,524820
Linseed oil	0,932	0,0337095	0,491591
Proof spirits of brandy	0,927	0,0335503	0,489268
Dry oak	0,915	0,0330946	0,489008
Olive oil	0,913	0,0330222	0,481569
Beech	0,854	0,0308883	0,450149
Dry elm, Dry ash	0,800	0,0289352	0,421966
Dry wainscot	0,747	0,0270182	0,394011
Dry yellow fir	0,657	0,0237630	0,346539
Cedar	0,613	0,0221715	0,323332
Dry white deal	0,569	0,0205801	0,300123
Cork	0,240	0,0186805	0,126590
Air	0,0012	0,0000434	0,000633

Case 1. The linear dimensions, or solidity of any body being given, to find its weight.

RULE.

Multiply the cubic inches contained in that body by the tabular weight corresponding; the product will give the weight, in pounds avoirdupois, or ounces troy.

EXAMPLES.

(1) What is the weight of a piece of oak of a rectangular form, whose solidity is 12096 cubic inches?

(2) What is the weight of a piece of fir, whose girt is 20 inches, and length 40 feet?

(3) What is the weight of an iron shot, of 7 inches diameter?

(4) What is the diameter of an iron shot, weighing 42 lb. avoirdupois?

(5) What is the weight of an iron bomb [illegible] of 3 inches thick, the greatest diameter being 16 inches?

(6) Required the weight of one of the Portland key-stones to the middle arch of Westminster Bridge, the diameter of the arch being 76 feet, the height of the key-stone 5 feet, the chord of its greatest breadth, to the front of the arch, 3 feet 4 inches, and its depth of the arch 4 feet.

(7) In the walls of Balbec, in Turkey, there are three stones, laid end to end, now in sight, that measure in length 61 yards; one of which in particular is 63 feet long, 12 feet thick, and ¾ yard over: Now if this block was marble, what power would balance it, so as to prepare it for moving?

Case 2. The weight of any body being given, to find the solidity and the specific gravity thereof.

RULE.

Divide the given weight by the tabular weight corresponding to the name of the same kind, and the quotient will be the solidity in cubic inches.

EXAMPLES.

(8) What will a block of marble, weighing 8 tons 14 cwt. come to, at 6*s*. per foot solid?

(9) Suppose that a man of war, with all its ordnance, rigging, and appointments, draws so much water as to displace 1300 tons of sea water, London beer measure. The weight of the vessel is required.

(10) What will a chain of standard gold weigh in water that raises a fluid an inch, in a vessel 3 inches square, when put into it; and supposing the workman had

adulterated the said chain with $14\frac{1}{2}$ ounces of silver, how much higher would the water, upon its immersion, be raised in the vessel?

(11) Hiero, king of Sicily, ordered his jeweller to make him a crown containing 63 ounces of gold; the workman thought of substituting part silver therein, to have a proper perquisite; which taking air, Archimedes was appointed to examine it; who, on putting it into a vessel of water, found it raised the fluid, or that itself contained 8,2245 cubic inches of metal; and having discovered that the cubic inch of gold more critically weighed 10,36 ounces, and that of silver but 5,85 ounces, he, by calculation, found what part of his majesty's gold had been changed; and you are desired to repeat the process.

PART V.

LXXVIII. CHRONOLOGY

Is the art of estimating and comparing together the times when any memorable transaction hath happened, such as related in history, whether civil or ecclesiastical.

It also takes a view of the various fasti, calendars, and methods of computing time, practised by different nations, compares them together, and settles such order and harmony among them, that the exact time in which any remarkable event happened may be certainly known.

	Years of the World.	Years before Christ.
Some have dated their events from the creation of the world - -	0000	4004
Others from the deluge or flood -	1656	2348
The Greeks from their olympiads, of 4 years each - - - -	3228	776
The Romans from the building of Rome	3251	753
The astronomers from Nabonassar, king of Babylon - - - -	3257	747
Some historians from the death of Alexander the Great - - -	3676	323
We from the birth of Christ - -	4004	A.D.
The Mohammedans from the flight of Mohammed, called the Hegira	4626	622

PROBLEM I.

To find whether any given year be leap year.

RULE.

Divide the given year by 4: if 0 remain, it is leap year; but if 1, 2, 3, remain, it is so many years after.

EXAMPLES.

(1) Is 1795 leap year?
(2) Is 1796 leap year?

Note 1.—Every fourth year is leap year, so called from leaping or advancing a day more that year than any other; that year has then 366 days in it, and February 29.

PROBLEM II.

To find the dominical letter till the year 1800.

RULE.

To the given year add its fourth part, omitting fractions; divide that sum by 7; the remainder taken from 7 leaves the index of the letter in the common year's reckoning.

1	2	3	4	5	6	7
A	B	C	D	E	F	G

But in leap years, this letter and its preceding one (in the retrograde order which these letters take) are the dominical letters.

EXAMPLES.

(3) For the year 1795, I demand the dominical letter.
(4) For the year 1796, I demand the dominical letter.

2. The dominical letter is that letter of the alphabet which points out in the calendar the Sundays throughout the year; thence also called the Sunday letter. Of these letters are consequently seven before mentioned, beginning with the first letter of the alphabet; and as in leap year there is an intercalary day, there are then two, one serving January and February, and its following letter the remaining part of the year.

PROBLEM III.

To know on what day in the week any proposed day of the month will fall.

RULE.

First find the dominical letter, then the day of the week the first of the proposed month falls on, which is known by the two following lines:

At Dover dwell George Brown, Esquire,
Good Christopher Finch, and David Frier.

Where the first letter of each word answers to the letter belonging to the first day of the months in order, from January to December.

3. You must observe that the 1st, 8th, 15th, 22d, and 29th day of any month fall on the same day of the week.

EXAMPLES.

(5) In 1795, on what day of the week does the 19th of May fall, it being queen Charlotte's birth-day?

(6) On what day of the week does the 4th of June fall in 1796, being king George the Third's birth-day?

PROBLEM IV.

To find the year of the solar, lunar, or golden number, and indiction cycles.

RULE.

To the given year add 9 for the solar, 1 for the lunar, 3 for the indiction; divide the sums in order by 28, 19, and 15: the remainder in each shows the year of its respective cycle.

EXAMPLE.

(7) Required the year of the solar, lunar, and indiction cycles, for the years 1795 and 1796.

4. The solar cycle, or the cycle of the sun, is a period of 28 years; in which time all the varieties of the

dominical letters will have happened; and the 29th year the cycle begins again, when the same order of the letters will return as were 28 years before.

At the birth of Christ, nine years had passed in this cycle.

The lunar cycle, or cycle of the moon, or golden number, is a period of 19 years, containing all the variations of the days on which the new and full moons happen; after which time they fall on the same days they did 19 years before, and she begins again with the sun.

But when a centesimal or hundredth year falls in the cycle, the new and full moon, according to the new style, will fall a day later than otherwise. The birth of Christ happened in the second year of this cycle.

The Roman indiction is a cycle of 15 years, which first began the third year before Christ.

PROBLEM V.

To find the epact till the year 1900.

RULE.

Multiply the golden number for the given year by 11; divide that product by 30, and from the remainder take 11, leaves the epact. If the remainder be less than 11, add 19 to it, and the sum will be the epact.

EXAMPLES.

(8) Find the epact for the year 1795.
(9) Required the epact for the year 1796.

5. The epact of any year is the moon's age, at the beginning of that year, that is, the days past since the last new moon.

PROBLEM VI.

To find the moon's age.

RULE.

To the epact add the number and day of the month; their sum, if under 30, is the moon's age. But if that sum is above 30, the excess in months of 31 days, or the excess

above 29 in a month of 30 days, shows the age or days since the last conjunction.

The moon's age taken from 30 leaves the day of the next new moon.

When the solar and lunar cycles begin together, the moon's age on the first of each month, or the monthly epacts, are called the numbers of the month, and are as follows; viz.

For	Jan.	Feb.	Mar.	April.	May.	June.
These	0.	2.	1.	2.	3.	4.
For	July.	Aug.	Sep.	Oct.	Nov.	Decem.
These	5.	6.	7.	8.	9.	10.

EXAMPLES.

(10) Required the moon's age on May 21, 1795.
(11) What is the moon's age on the 24th of March, 1796?

6. The moon's age is = how many days are past since the day of her change, which age never exceeds 30 days.

PROBLEM VII.

To find when Easter-day will happen.

RULE.

Find on what day of March the new moon falls nearest to the 21st in common years, or nearest the 20th in leap years: then the Sunday next after the full, or 15th day of that new moon, will be Easter-day.

If the 15th day fall on a Sunday, the next Sunday is Easter-day.

EXAMPLES.

(12) On what day does Easter-Sunday fall for the year 1795?
(13) Required the time of Easter-day for the year 1796.

7. Easter is the time when Christians celebrate the resurrection of Christ from the grave, and took its rise from Eastra, the name of the Saxon deity or goddess whose festival was celebrated about this time of the year; and after its abolishment by Christianity, the name was re-

tained, and is to this day used to signify the festival of Christ's resurrection, as mentioned above.

PROBLEM VIII.

To find the time of the moon's southing.

RULE.

Multiply the moon's age by 4; divide the product by 5, quotes the hours; and the remainder, multiplied by 12, gives the additional minutes.

If this time be less than 12 hours, it is the time of southing after mid-day; but if greater, 12 hours taken from it leaves the southing after midnight.

EXAMPLES.

(14) Required the time of the moon's southing at London on the 21st of May, 1795.

(15) At what time does the moon come to the meridian at Bristol quay, on March 24, 1796?

8. The moon's southing at any place is the time when she comes to the meridian, or is full south of that place, which is every day later, by about 48 minutes; occasioned by the hours in a day being divided by the thirty times she passes the meridian from new moon to new moon.

PROBLEM IX.

To find the time of high water at any place.

RULE.

To the time of the moon's southing add the time the moon has passed the meridian, to make high water at that place, and the sum will show the time of high water.

The distance of the moon from the meridian, when high water at the following places, is: At London, ☽ bears N. E. or S. E. 3 h. 0 m. Bristol quay, ☽ bears E. by S. and W. by N. 6 h. 45 m.

EXAMPLES.

(16) On the 21st of May, 1795, at what time is it high water at London?

(17) On the 24th of March, 1796, at what time is it high water at Bristol quay?

9. High water is the state of the tide when highest, or the time it ceases to flow up.

QUESTIONS *for Exercise at leisure Hours.*

(1) England was conquered by William I. October 4, 1066; his son, William II., came to the crown Sept. 8, 1087, and left it August 2, 1100; William III. received it Feb. 3, 1689, and died March 8, 1701. How many days did each of these princes govern, respect being had to the intercalary days, and to February every leap year, as they rose in the course of time?

(2) Richard I. succeeded his father Henry II. July 7, 1189; John, his brother, succeeded him April 6, 1199; Richard II. succeeded Edward III. on the 21st of June, 1377, and was deposed by Henry IV. on the 30th of September, 1399; Richard III. caused his nephew, Edward V., and his brother, to be murdered on the 18th of June, 1483, and was slain himself on the 22d August, 1485. How many days was the realm governed by the three Richards, respect being still had to the intercalary days as they happened?

(3) The first Queen Mary came to the crown July 8, 1553; she reigned 5 years 4 months and 9 days; her sister Elizabeth succeeded, and James I. came to the throne the 14th of March, 1602, who left it to his son, Charles I., on the 27th of March, 1625, who was forced from it January 30, 1648. The question is, How many days did these princes reign? and at the death of Charles I. how long had England been under an uninterrupted succession of Protestant princes (Mary the first being the last professed papist that enjoyed the crown), not neglecting the intercalary days in February as before?

(4) A grant was made February 14, in the 10th of Henry I., who began his reign August 2, 1100; it was resumed

November 19, in the 4th of Henry III., who came to the crown October 19, 1215; it was revived the 16th day of July, in the 13th of Henry VII., who ascended the throne August 22, 1486; but it was a second time revoked, and finally suppressed, in the 16th of his successor, Henry VIII., on the 10th of May. Now, as this man's father died July 21, 1509, the question is, How many days was this grant in force, and how many did it lie dormant?

LXXIX. GEOGRAPHY.

Examples on the Terrestrial Globe, or Maps.

(1) WHAT is the latitude and longitude of Pekin, in China, and Cape Horn?

(2) Required the name of that place whose latitude is 18° N. and longitude 76½° W.; also of another place, whose latitude is 34½ S. and longitude 16½ E.

(3) What is the difference of latitude between London and Naples; also between the island of Barbadoes, and the Cape of Good Hope?

(4) Required the distance (in English miles) Jamaica is from London; also the names of all those places that are at the same distance from London as Rome is.

(5) Required the sun's declination, right ascension, and meridian altitude, on the 20th of May.

(6) Required the time of the sun's rising and setting, on the 20th of May; also his amplitude at the same time; likewise when the twilight begins and ends.

(7) What is the sun's azimuth and altitude, on the 20th of May, at 4 o'clock in the afternoon?

(8) What is the angle of position, or bearing, of Port Royal in Jamaica from London; and, on the contrary, London from Port Royal?

(9) When it is noon, or twelve o'clock, at London, what o'clock is it at Pekin in China? Also at what places are they breakfasting, dining, and supping, suppose they breakfast at 7 o'clock, dine at 1, and sup at a quarter after 9?

(10) What places are those to which the sun is vertical on the 2d of May?

(11) On what two days of the year will the sun be vertical to Candy in the isle of Ceylon?

(12) What places are those in the north frigid zone, on which the sun begins to shine constantly, without setting, on the 20th of May?

(13) On what day does the sun begin to shine constantly, without setting, at the Cherry island, whose lat. is 74° north; and how long?

(14) What places are those to which the sun is rising, setting, or in the meridian; also those places which are enlightened, and those which are not; on the 20th of May, at 8 in the morning?

(15) By the almanack for this present year, on the ——— at ——— will happen an eclipse of the sun. I demand to know to what part of the globe the same will be visible.

(16) On the 3d of June, 1769, in the afternoon, happened a transit of Venus over the sun; the beginning of this transit was at 7 h. 13 m., middle 10 h. 35 m., end 1 h. 55 m. I demand to know where the beginning, middle, and end thereof were visible.

(17) In what latitude is the longest day 20 hours long?

(18) What inhabitants of the earth are those called Antœci, Periœci, and Antipodes, with respect to London?

(19) What is the sun's declination on the 20th of June by the analemma?

(20) It is required to find, by the analemma, the sun's place in the ecliptic, and his right ascension, on the 12th of May.

(21) Required to find, by the analemma, the time of the sun's rising and setting, with his amplitude, on the 30th of March.

(22) What is the sun's altitude and azimuth for the 26th of April, at 10 in the morning, by the analemma?

(23) On the 24th of April in the morning, in lat. 51° 30′ N. the sun's altitude was 26°; required the hour and azimuth, by the analemma.

(24) Suppose a ship sails from a port A, in lat. 38°, to another port B, in lat. 5° N. and then finds her difference of longitude 43°: required her course, and distance sailed.

(25) A ship sails from a port A, in lat. 26° N. to another port B, in lat. 20° S. upon a course of 46°. Required the difference of longitude, and distance sailed.

(26) Suppose a ship sails from a port A, in lat. 51° 30′, to another port B, in lat. 18°, distance 2226 miles: required the difference of longitude, and angle of the course.

(27) Suppose a ship sails from a place A, in lat. 51°, on a course making an angle with the meridian of 40°, till the difference of longitude be found to be 20°: required the difference of latitude, and distance sailed.

(28) A ship from the latitude 47° 30′ N. has sailed S. W. by S. 1080 miles. Required the difference of latitude and longitude.

Examples on the Celestial Globe.

(1) Required the time of the sun's rising and setting, also the beginning and end of the crepusculum, or twilight, on the 21st of June.

(2) What is the moon's diurnal motion in the ecliptic, also at what time does she rise, set, and come to the meridian, on the 20th of May?

(3) Required the latitude of the Moon, and her declination, on the 20th of May?

(4) At what time does the planet Jupiter rise, culminate, and set, on the 20th of May? Also, what is its right ascension, declination, amplitude, and the azimuth, on the above day?

(5) What is the right ascension, declination, latitude, and longitude, of Pollux?

(6) What star is that whose right ascension is 65° 30′, and its declination 12° 15′ 30″ north? Also what time does it rise, come to the meridian, set, and what is its amplitude, on the 20th of July, in the lat. of London?

(7) On what day of the year will the star Arcturus rise and set cosmically at London?

(8) Required the time when Procyon and Canis Minor will rise and set acronically at London.

(9) On what day of the year will Altayr culminate, or come to the meridian, with the sun?

(10) At what time of the year will the Pleiades, or Seven Stars, be upon the meridian at midnight?

(11) What is the oblique ascension of Sirius, and what is the time of its continuance above the horizon of London?

(12) What is the altitude and azimuth of Capella, on the

20th of May, at 10 o'clock at night, in the latitude of London?

(13) The altitude of Cor Leonis, on the 22d of May, at London, was 20°; required the hour of the night.

(14) A person being in a certain place, on the 20th of May, at ¼ after 3 in the morning, observed the Pleiades were then rising. Required the latitude of the place of observation.

(15) On the 11th of May, in the latitude 51° 30′, the two stars, Luci a Lyræ, and Altayr, will be both on the same azimuth. Required the hour of the night.

(16) On the 11th of May (lat. as before) the bright star marked in Pegasus's wing, and that in the head of Andromeda, will both have an equal altitude. Required the hour.

(17) A person being at sea, found, by observation, that Sirius was then upon the meridian, and Arcturus rising: required the lat. of the place of observation.

(18) Another person being at a certain place, found, by observation, Cor Hydræ and Procyon both on the azimuth of 78° 45′ S. E. one with 5° of altitude, and the other with 35°. Required the latitude of the place of observation.

(19) To what latitude south must I travel, to lose sight of the star Capella?

(20) Represent the face of the heavens on the globe on the 20th of May, at 10 at night.

(21) By an observation made at Jamaica, of a comet, on the 31st of March, 1759, at 5 o'clock in the morning, its altitude was found to be 22° 50′, and azimuth 71° South East. Another observation was made at London on the 6th of May, 1759, at 10 at night, of the same comet, and then its altitude was found to be 16°, and its azimuth 37° S. W. It is required to know the place of the comet at each observation.

(22) Required the time of the above comet's rising, southing, and setting, at London, on the 31st of March, 1759; also its latitude, longitude, declination, and ascension.

(23) Required the apparent path, among the fixed stars in the heavens, of the above comet, also its velocity.

Note.—These problems are answered by Mr. Hill's twelve-inch globes.

PART VI.

LXXX. ALGEBRA.

ALGEBRA is a kind of specious arithmetic, or an arithmetic in letters; and is that science which teaches, in a general manner, the comparison of abstract quantities; by means whereof such questions are resolved whose solutions would be sought in vain from common arithmetic.

Here every quantity, whether given or required, is commonly represented by some letter of the alphabet; the known or given quantities, for distinction's sake, being noted by the first letters, *a*, *b*, *c*, *d*, &c. and the unknown ones by the last letters, *x*, *y*, *z*, &c.

There are, moreover, in algebra, certain signs or notes, made use of to show the relation and dependence of quantities one upon another, whose signification the learner ought first of all to be made acquainted with. (See the characters for abbreviation, next before page 1.)

LXXXI. ADDITION.

ADDITION in Algebra is performed by connecting the quantities by their proper signs, and joining in any sum such as can be united.

For performing which, observe the following

RULE.

1. If the quantities to be added are alike, and have the same sign, add the coefficients together, and to their sum prefix the common sign, and subjoin the common letter or letters.

2. If the quantities to be added are alike, but have unlike signs, add together the coefficients of the affirmative terms (if there be more than one), and do the same by the negative ones; and to their difference prefix the sign of the greater, adding the common letter or letters.

3. If the quantities to be added are unlike, write them down one after the other, with their proper signs and coefficients prefixed.

EXAMPLES.

(1) To $6a+7b-3c$
Add $10a+b-7c$

(2) $ab-6b+4x+10y-15z+6$
$6ab-b+x+4y-9z+3$

(3) To $-6b-7c-8x$
Add $4b+9c+5x$

(4) $6a-6x+7y-10$
$-6a+6x-4y+13$

(5) To $4a+6b+4c-6$
Add $4x-7y+4z$

(6) $2a-6bc$
$6x+10$

(7) To $10a\sqrt{bc}$
Add $6a\sqrt{bc}$

(8) $a\sqrt{bb+cc}$
$4\sqrt{bb+cc}$

(9) $b+3\sqrt{aa-u}$
$b-3\sqrt{aa-u}$

LXXXII. SUBTRACTION.

SUBTRACTION of algebra is performed by the following general

RULE.

Change the signs of the quantity to be subtracted into their contrary signs, and then add it, so changed, to the

quantity from which it was subtracted (by the rule of addition): the sum arising will be the remainder.

EXAMPLES.

(1) From $3a$
$-3a$

(2) $2a-4x+7y-7$
$6a+4x+7y+4$

(3) $6b-4c+4x$
$7b+7c-9x$

(4) From $10a\sqrt{bc}$
$6a\sqrt{bc}$

(5) $6b\sqrt{aa+xx}$
$4b\sqrt{aa+xx}$

(6) $b+a$
$a-\sqrt{cc-a}$

LXXXIII. MULTIPLICATION.

MULTIPLICATION of algebra is also performed by the following general

RULE.

Multiply the coefficients (if any) together, as in Sect. 4. and to their product join the letters, and prefix the proper sign before them, which, when the signs of the factors are alike, that is, both $+$, or both $-$, the sign of the product is $+$; but when the signs of the factors are unlike, the sign of the product is $-$.

EXAMPLES.

(1) Mul. $a+c$
By b

(2) $-a-b-c$
$-d$

(3) $x+y+z$
a

(4) Mul. $-8x$
By $-4a$

(5) $12x+6y$
$4a$

(6) $-6d$
$+7b$

(7) Mul. $4x-5y+z$
By $6f$

(8) $2a-4b$
$2a+4b$

(9) $aa+ab+bb$
$a-b$

(10) Mul. $xx-ax$
By $x+a$

(11) $xx+xy+yy$
$xx-xy+yy$

(12) Mul. $aaa-3aab+3abb-bbb$
By $aa-2ab+bb$

(13) $\sqrt{bc}+dcc$
$\sqrt{ac}$

(14) Mul. $\sqrt{xx}-zz$
By $\sqrt{xx}-zz$

(15) $6cd\sqrt{b}+ad$
$3a\sqrt{ca}$

LXXXIV. DIVISION.

DIVISION of algebraic quantities is the direct contrary to that of multiplication, and consequently performed by direct contrary operations.

RULES.

1. When the quantities in the dividend have like signs to those in the divisor, and no coefficient in either, cast off all the quantities in the dividend that are like those in the divisor, and set down the other quantities with the sign + for the quotient.

2. When the quantities in the dividend have unlike signs to those in the divisor, then set down the quotient quantities, found as in the last rule, with the sign — before them.

3. If the quantities in the divisor cannot be exactly found in the dividend, then set them both down like a vulgar fraction, and find all the quantities of the same letters that are in the dividend and divisor, and proceed with the co-efficient as in Case 1, Sect. 38.

4. If the quantity to be divided is compound, range its parts according to the dimensions of some one of its letters, and proceed as in Sect. 5.

5. Different powers or roots of the same quantity are divided by subtracting the exponent of the divisor from that of the dividend, and placing the remainder as an exponent to the quantity given.

EXAMPLES.

Divisor. Dividend.

(1) $d)ad+6d($ (2) $-d)-ad-bd($ (3) $a)aa+ab($

(4) $-a)ab($ (5) $b)+ab-bd($ (6) $-bc)\ abc+bcd-bcf($

(7) $7b)\ 42-db($ (8) $2bx)8abx-18bxc($ (9) $2b)ab-bb($

(10) $20a)10ab-15ac($

(11) $a-b)aaa-3aab+3abb-bbb($

(12) $a+b)aa+2ab+bb($ (13) $a+b)aa-bb($

(14) $3a-6)6a-96($

(15) $3x^2-4x+5)\ 18x^4-45x^3+82x^2-67x+40($

(16) $4z-5a)48x^3 76ax^2-64a^2x+105a^3($

(17) $3x+4a)81x^4-256a^4($

(18) $2x-3a)16^4x-72a^2x^2-81a^4($

(19) $2xy\sqrt{z})4xy\sqrt{xzz}($

(20) $20\sqrt{2cy})\ 60ab\sqrt{10acxy}($

(21) $x^2)\ x^5($ (22) $\overline{a+x}|^4)\overline{a+x}|($

LXXXV. FRACTIONS.

REDUCTION of algebraic fractions is of the same nature, and requires the same management, as that of numbers.

A mixed quantity is reduced to an improper fraction by the rules in Sect. 38, Case 3.

EXAMPLES.

(1) Reduce $a-x+\frac{a^2-ax}{x}$ to an improper fraction.

(2) Reduce $a+b+\frac{x}{2}$ to an improper fraction.

(3) Reduce $a-x+\frac{aa-ax}{x}$ to an improper fraction.

An improper fraction is reduced to a mixed quantity, by the rule in Sect. 38, Case 4.

EXAMPLES.

(4) Reduce $\frac{a^2-x^2}{x}$ to a mixed quantity.

(5) Reduce $\frac{az+bz+x}{z}$ to a mixed quantity.

(6) Reduce $\frac{ax-xx+aa-ax}{x}$ to a mixed quantity.

Fractions of different denominations are reduced to fractions of equal value, and to have the same denominator, by the rule in Sect. 38, Case 5.

EXAMPLES.

(7) Reduce $\frac{a}{b}$, $\frac{c}{d}$, and $\frac{e}{f}$, to a common denominator.

(8) Reduce $\frac{b+c}{a+b}$ and $\frac{d-c}{b-d}$ into one denomination.

Fractional quantities are reduced into their lowest terms by the rule in Sect. 38, Case 1.

EXAMPLES.

(9) Reduce $\frac{aaa-abb}{aa+aab+bb}$ to its lowest terms.

(10) Reduce $\frac{25az}{5az+15az}$, and $\frac{aaa+bbb}{aa-bb}$, to their lowest terms.

The rules for addition, subtraction, multiplication, and division of algebraic fractions, are the same as for numerical fractions; see Sect. 38, 39, 40, and 41.

EXAMPLES in ADDITION.

(1) Add $\frac{a}{b}+\frac{c}{d}$ and $\frac{d}{e}$ into one sum.

(2) Add $\frac{a-b+d}{d+a}$ and $\frac{a+b-d}{d+a}$ together.

(3) Add $\frac{2a-b}{d+c}$ and $\frac{2b-a}{a+c}$ together.

(4) Add $\frac{a+b}{d}$ to $\frac{2a+c}{d}$.

EXAMPLES in SUBTRACTION.

(1) From $\frac{x}{2}$ take $\frac{x}{3}$. (2) From $\frac{a+x}{b}$ take $\frac{a-x}{c}$.

(3) From $\frac{bb+aa}{c}$ take $\frac{bb}{c}$. (4) From $\frac{2b}{a+d}$ take $\frac{a+b-d}{a+d}$.

EXAMPLES in MULTIPLICATION.

(1) Mul. $\frac{a+b}{c}$ by $\frac{a-b}{d}$. (2) Mul. $a+\frac{b}{c}$ by $\frac{d}{c}$.

(3) Mul. $\frac{3a-2b}{2d+c}$ by $\frac{4a+2b}{d}$.

(4) Mul. $2a+\frac{b}{c}-25$ by $36+4c$.

EXAMPLES in DIVISION.

(1) Divide $\frac{acd+d}{cd}$ by $\frac{d}{c}$. (2) Divide $\frac{a-b}{a}$ by $\frac{a+b}{a-b}$.

(3) Divide $a+\frac{b}{c}$ by $d+\frac{e}{f}$. (4) Divide $\frac{ab}{c}$ by $\frac{1}{c}$.

LXXXVI. INVOLUTION.

INVOLUTION is the raising of any given quantity to any proposed power.

1. If the quantity proposed to be involved has no index, that is, if it be not itself a power or a surd, the power thereof will be represented by the same quantity under the given index or exponent.

Thus, the cube or third power of x is expressed by x^3.

And the sixth power, $a+z$, by $\overline{a+z}|^6$, &c.

2. But if the quantity proposed be itself a power or surd, it will be involved by multiplying its exponent by the exponent of the proposed power.

Thus, the fifth power of x^2 is x^{10}, the fourth power of $\overline{ax+y}|^3$ is $\overline{ax+y}|^{12}$, the third power of $\overline{a-x}|^{\frac{1}{2}}$ is $\overline{a-x}|^{\frac{3}{2}}$.

3. A quantity composed of several factors multiplied together is involved by raising each factor to the power proposed.

Thus, the square or second power of ax is a^2x^2, the cube

or third power of $2ax$ is $8a^3 8x^3$; the fourth power of $4 \times \overline{aa - xx} \times \overline{a + b + c}$ is $16 \times \overline{aa - xx}|^4 \times \overline{a + b + c}|^4$, and the square of the radical quantity $a^{\frac{1}{2}} \times a + x^{\frac{1}{3}}$ is $a \times \overline{a + x}|^{\frac{2}{3}}$.

4. A fraction is involved by raising both the numerator and the denominator to the power proposed.

Thus, the cube or third power of $\frac{a}{b}$ is $\frac{aaa}{bbb}$, and the fourth power of $\frac{2a^2x}{3b^2}$ is $\frac{8a^8x^4}{81b^8}$; likewise the sixth power of $\frac{\overline{aa+xx}|^{\frac{1}{2}}}{\overline{a-x}|^2}$ is $\frac{\overline{aa+x}|^3}{\overline{a-x}|^2}$

5. Quantities compounded of several terms are involved by a continual multiplication of all their parts.

Thus $a + b \times a + b = a^2 + 2ab + b^2$.

EXAMPLES.

(1) Involve or raise x to the fourth power.

(2) Raise $\overline{ax+z}|^2$ to the fifth power.

(3) Involve $3x^2z^2$ to the third power.

(4) Involve $\frac{5ab}{2c}$ to the sixth power.

(5) Involve or raise $a+b$ to the sixth power. This is called a binomial root.

(6) Involve or raise $a-b$ to the sixth power.

(7) Involve or raise $a-b$ to the sixth power. This is called a residual root.

There is a rule or theorem, given by Sir Isaac Newton, whereby any power of a binomial, or $x-y$, may be expressed in simple terms, without the trouble of those tedious multiplications which are required otherwise.

THEO. 1. $\times \frac{m-0}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \times \frac{m-4}{5} \times \frac{m-5}{6} \times$ &c.

Note, m is the exponent of the power, that is, $m=7$ in the seventh power, 6 in the sixth power, &c.

So that if $x-y$ is to be raised to any power m, the terms, without their coefficients, will be

$$x^{m}, x^{m-1}, x^{m-2}y, x^{m-3}y, x^{m-4}yx, x^{m-5}yx, x^{m-6}y,$$

&c. continued till the exponent of y becomes equal to m: and the coefficients of the respective terms will be

$$1, m\ m \times \frac{m-1}{2} m \times \frac{m-1}{2} \times \frac{m-2}{3} m, \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4},$$

$$m \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \times \frac{m-4}{5} m, \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4}$$

$$\frac{m-4}{5} \times \frac{m-5}{6} \times \frac{m-6}{xy6} +, \text{ \&c.}$$

So by this theorem any quantity, consisting of two terms, is raised to any power m, with great ease and perspicuity, and will be of great service to the young algebraist, if properly demonstrated to him by his tutor.

LXXXVII. EVOLUTION.

EVOLUTION, or the extraction of roots, being directly the contrary to involution, or raising of powers, is performed by converse operations, *viz.* by the division of indices, as involution was by their multiplication.

Thus, the square root of x^6 is x^3, the cube root of x^6 is x^2, also the biquadratic root of $\overline{x+y}|^8$ will be $\overline{x+y}|^2$: and the cube root of $\overline{xx-yy}|^6$ will be $\overline{xx-yy}|^2$. Moreover the square root of $\overline{xx-yy}|^{\frac{1}{2}}$ will be $\overline{xx-yy}|^{\frac{1}{4}}$, its cube root $\overline{xx-yy}|^{\frac{1}{6}}$, and its biquadratic root $\overline{xx+yy}|^{\frac{1}{8}}$, and so of others.

Evolution of compound quantities is performed by the following

RULE.

First, place the several terms, whereof the given quantity is composed, in order, according to the dimensions of some letter therein, as shall be judged most commodious: then let the root of the first term be found, and placed in the quotient; which term being subtracted, let the first term of the remainder be brought down, and divided by twice the first term of the quotient, or by three times its square, or four times its cube, &c. according as the root to be extracted is a square, cubic, or biquadratic one, &c. and let the quantity thence

arising be also written down upon the quotient; and the whole be raised to second, third, or fourth, &c. power, according to the aforesaid cases respectively, and subtracted from the given quantity; and if any thing remain, let the operation be repeated, by always dividing the first term of the remainder by the same divisor, found as above.

EXAMPLES.

(1) It is required to extract the square root of $x^2+2xy+y^2$.
(2) It is required to extract the square root of $x^2-2xy+y^2$.
(3) It is required to extract the square root of $x^4-2x^3y+3x^2y^2-2xy^3+y^4$.
(4) Extract the cube root of $x^3-6x^2y+12xy^2+8y^3$.
(5) Extract the biquadratic root of $16x^2-96x^3y+216x^2y^2 216xy^3+81y^4$.

LXXXVIII. INVOLUTION of SURD QUANTITIES.

1. When the surds are not joined to rational quantities, they are involved to the same height as their index denotes by taking away their radical sign.

Thus $\sqrt{xx}$ will be x^2, and $\sqrt{xx+yy}$ will be x^2+y^2, &c.

2. When surds are joined to rational quantities, involve the rational quantities to the same height as the index of the surd denotes; then multiply the involved quantities into the surd quantities, after the radical sign is taken away, as before.

Thus $x\sqrt{yy}$ will be x^2y^2, and $4x\sqrt{xx+yy}$, will become $16x^4+16x^2y^2$; likewise, $2x^3\sqrt{x+y^2}$ will become $8x^5+8x^2y^2$, &c.

LXXXIX. EQUATIONS.

AN equation is when two equal quantities, differently expressed, are compared together, by means of the sign = placed between them.

REDUCTION of SINGLE EQUATIONS.

RULES.

1. Any term of an equation may be transposed to the contrary side, if its sign be changed.
   Thus, $x+12=20$, then will $x=20-12=8$.

2. If there be any quantity by which all the terms of an equation are multiplied, let them all be divided by that quantity: but if all of them be divided by any quantity, let the common divisor be cast away.

Thus, $ax=ab$, then will $x=\frac{b}{a}$; also, if $\frac{x}{b}=\frac{a}{b}$, $x=a$, by the latter part of the rule.

3. If there be irreducible fractions, let the whole equation be multiplied by the product of all their denominators; or, which is the same, let the numerator of every term in the equation be multiplied by all the denominators except its own, supposing such terms (if any there be) that stand without a denominator to have a unit subscribed.

Thus $x+\frac{x}{2}+\frac{x}{3}=11$, reduced is $6x+3x+2x=66$, or $x=6$ per rule 5. Again, $\frac{2x}{3}+12=\frac{4x}{5}+6$, this reduced will become $10x+180=12x+90$: then per rule 1, $x=45$.

4. If in your equation there be an irreducible surd, wherein the unknown quantity enters, let all the other terms be transposed to the contrary side (by rule 1.), and then if both sides be involved to the power denominated by the surds, an equation will arise free from radical quantities, unless there happen to be more surds than one; in which case the operation is to be repeated.

Thus $\sqrt{x}+4=12$, by transposition, becomes $\sqrt{x}=12-4=8$; which, by squaring both sides, gives $x=64$.

So likewise, $\sqrt{aa+xx}-c=b$, becomes $\sqrt{aa+xx}=b+c$; squared, gives $aa-xx=bb+2cb+cc+$, then per rule 1. $x^2=a^2+b^2+2cd+c^2$, and $x=\sqrt{a^2+b+2cd+c^2}$.

5. Having by the preceding rules, if there be occasion, cleared your equation of fractional and radical quantities, and so ordered it, by transposition, that all the terms wherein the known quantities are found, may stand on the same side thereof, let the whole be divided by the coefficients, or the sum of the coefficients, of the highest power of the said unknown quantity.

Thus, if $6x=24$, then will $x=\frac{24}{6}=4$; and if $4x=48-2x$, then will $6x=48$, per rule 1, and $x=\frac{48}{6}=8$.

EXAMPLES,

For the learner's exercise in the foregoing rules, set down promiscuously.

(1) If $20-3x-8=60-7x$, what is the value of x?

(2) When $5x-16=3x+12$, what is x?

(3) If $\frac{3x}{4}+5=\frac{5x}{6}+2$, what is x equal to?

(4) If $\frac{7x}{8}-5=\frac{9x}{10}-8$, what is x?

(5) When $\frac{5x}{9}-8=74-\frac{7x}{12}$, query x?

(6) If $56-\frac{3x}{4}=48-\frac{5x}{8}$, what is the value of x?

(7) Required the value of x, when $36-\frac{4x}{9}=8$.

(8) When $\frac{2x}{3}=\frac{176-4x}{5}$, query x?

(9) If $\frac{45}{2x+3}=\frac{57}{4x-5}$, what is the value of x?

(10) If $\frac{42x}{x-2}=\frac{35x}{x-3}$, what is x equal to?

(11) If $\frac{xx-12}{3}=\frac{x-4}{4}$, what is x equal to?

(12) When $\frac{5xx}{16}=8+12$, what is the value of x?

(13) Suppose $\frac{x+1}{2}+\frac{x+2}{3}=16-\frac{x+3}{4}$, query x?

(14) Suppose $ax+b^2=\frac{ax^2+ac^2}{a+x}$, query x?

(15) If $\sqrt{\frac{5x}{3}}+12=17$, what is x?

(16) What is the value of x, when $\sqrt{12+x}=2+\sqrt{x}$?

(17) If $\sqrt{x}+\sqrt{a+x}=\sqrt{\frac{2a}{a+x}}$, what is x?

(18) Suppose $615x-7xxx=48x$, query x?

(19) Suppose $\sqrt{a^2+x^2}=\sqrt{b^4+x^4}$, what is x equal to?

LXXXIX. Of the EXTERMINATION of UNKNOWN QUANTITIES; or, the REDUCTION of two or more EQUATIONS to a single one.

RULE.

1. Observe which of all your unknown quantities is the least involved, and let the value of that quantity be found in each equation (by the rules already given), looking upon all the rest as known: let the values thus found be put equal to each other (for they are equal), because they all express the same thing; whence new equations will arise, out of which that quantity will be totally excluded; with which new equations the operations may be repeated, and the unknown quantities exterminated, one by one, till at last you come to an equation containing only one unknown quantity.

Thus, let the given equations be $x+y=12$, and $5x+3y=50$, to find x and y.

Now by transposing y and $3y$ we get $x=12-y$, and $5x=50-3y$, from the last of which equations, $x=\frac{50-5y}{5}$.

Now by equating these two values of x, we have $12-y=$

$\frac{50-3y}{5}$ and therefore $60-5y=50-3y$, from which $y=\frac{10}{2}$, $=5$, and $x=12-y=12-5=7$.

2. Or, let the value of the unknown quantity, which you would first exterminate, be found in that equation wherein it is the least involved, considering all the other quantities as known; and let this value and its powers be substituted for that quantity and its respective powers in the other equation; and with the new equations thus arising repeat the operation, till you have only one unknown quantity and one equation.

Thus x being the first equation (in the last example), $=12-y$, then by substituting this value of x, in the second, that is $60-5y$, must be wrote in the room of its equal $5x$; whence will be had, $60-5y+=50$;
and from hence $y=\frac{10}{2}=5$, as before.

3. Or, lastly, let the given equations be multiplied or divided by such numbers or quantities, whether known or unknown, that the term which involves the highest power of the unknown quantity to be exterminated may be the same in each equation; and then, by adding or subtracting the equation, as occasion shall require, that term shall vanish, and a new equation emerge, wherein the number of dimensions (if not the number of unknown quantities) will be diminished.

By multiplying the first equation by 5, we shall have - - - - $5x+5y=60$
from whence subtracting the 2d equation, viz. $5x+3y=50$
there remains - - - $2y=10$
whence $y=5$, and x by the first or second equation will be 7, still the same as before.

The first of these three ways is the most commonly used; but the last of them is, for the general part, the most easy and expeditious in practice.

EXAMPLES.

(1) Let $\left\{\begin{array}{l} 5x+8y=106 \\ 4x-5y=\ \ 5 \end{array}\right\}$ query, the value of x and y?

(2) Let $\left\{\begin{array}{l} 5x-3y=150 \\ 10x+15y=825, \end{array}\right\}$ query, x and y?

(3) Let $\left\{\begin{array}{l} \frac{x}{2}+\frac{y}{3}+\frac{z}{4}=62 \\ \frac{x}{3}+\frac{y}{4}+\frac{z}{5}=47 \\ \frac{x}{4}+\frac{y}{5}\ \ \frac{z}{6}=38 \end{array}\right\}$ query x, y, and z?

(4) Let $\left\{\begin{array}{l} \frac{x}{4}+\frac{y}{5}=15 \\ \frac{x}{6}+\frac{y}{9}=9 \end{array}\right\}$ query x and y?

(5) Given $\left\{\begin{array}{l} \frac{x}{2}-12=\frac{y}{4}+8 \\ \frac{x+y}{5}+\frac{x}{3}-8=\frac{2y-x}{4}+27 \end{array}\right\}$ what is the value of x and y?

(6) Given $\left\{\begin{array}{l} x+y=80 \\ x+z=70 \\ y+x=60 \end{array}\right\}$ to find x, y, and z.

(7) Suppose $\left\{\begin{array}{l} x+100=y+z \\ y+100=2x+2z \\ z+100=3x+3y \end{array}\right\}$ what is x, y, and z equal to?

(8) Let there be given $x-y=2$, $xy+5x-6y=120$, to exterminate x?

(9) Let $\left\{\begin{array}{l} x+y=s \\ x^2-y^2=d \end{array}\right\}$ query x, y, and z?

(10) Let $\left\{\begin{array}{l} x+y+z=12 \\ x+2y+3z=20 \\ \frac{x}{3}+\frac{y}{2}+z=6 \end{array}\right\}$ query x, y, and z?

XC. QUADRATIC EQUATIONS.

A QUADRATIC equation is when it involves one unknown quantity, and at the same time the square of that quantity, and the product of it multiplied by some unknown quantity.

Of these equations there are three forms:

$xx+6x+12=52$ the first form.
$2xx-6x+12=20$ second.
$6xx-x=8$ third.

All of which may be solved by the following

RULES.

1. Transpose all the terms that involve the unknown quantity to one side, and the known terms to the other side of the equation.
2. If the square of the unknown quantity be multiplied by any coefficient, you are to divide all the terms by that coefficient, that the coefficient of the square of the unknown quantity may be unit.
3. Add to both sides the square of half the coefficient prefixed to the unknown quantity itself; and the side of the equation that involves the unknown quantity will then be a complete square.
4. Extract the square root from both sides of the equation, which you will find on one side always to be the unknown quantity, with half the aforesaid coefficient subjoined to it; so that by transposing this half you may obtain the value of the unknown quantity expressed in the known term.

Thus, by Rule 1, the three aforesaid equations will become as follow;

First $xx+6x=52-12=40$;
Second $2x^2-6x=20-12=8$;
And Third $x^2-6x=-8$.

And by Rule 2, the second equation will become

$xx-\frac{6x}{2}=\frac{8}{2}$, consequently $x^2-3x=4$.

Then, by Rule 3, these three equations will become as follows, viz.

First $x^2+6x+9=40+9=49$;
Second $x^2-3x+2,25=4+2,25=6,25$;
And Third $x^2-6x+9=9-8=1$.

Also, by Rule 4, they will become as follow:

First $x+3=\sqrt{49}=7$;
Second $x-1,5=\sqrt{6,25}=2,5$;
And Third $x-3=\sqrt{1}=1$.

Then by the rules of reduction,

$$\left.\begin{array}{l} x=7-3=4 \\ x=2,5+1,5=4 \\ x=1+3=4 \end{array}\right\} \text{so, } x=4.$$

All quadratic equations may be solved by the following general theorem:

Thus, suppose the second equation were required to be solved;
First let $A=2$, $B=6$, and $C=8$.
Then it will stand $Axx-Bx=C$.
Per Rule 2, $xx-\frac{Bx}{A}=\frac{C}{A}$.

Also per Rule 3, $xx=\frac{bx}{a}+\frac{bb}{4aa}+\frac{bb}{4aa}+\frac{c}{a}$; but the two ractions $\frac{bb}{4aa}$ and $\frac{c}{a}$ when thrown into one, give $\frac{abb+4aac}{4aaa}$, which, divided by a, give $\frac{bb+4ac}{4aa}$; therefore $xx-\frac{bx}{a}+\frac{4aa}{bb}+\frac{bb+4ac}{4aa}$.

Now let $bb+4ac=ss$, then the equation will stand thus, $bx-\frac{ax}{a}+\frac{bb}{4aa}=\frac{ss}{4aa}$; then per Rule 4, $x-\frac{2a}{b}=\frac{s}{2a}$; therefore $x-\frac{b+s}{2a}$; that is, $x=\frac{b+s}{2a}$, or $x=\frac{b+s}{2a}$.

Q. E. F.

EXAMPLES.

(1) Suppose $xx-4x=32$, what is the value of x?
(2) Suppose $12xx-420x=-1200$, query x?
(3) Suppose $4x+60x=216$, what is x equal to?

Suppose $a=21$, or any other number, and $e=7$, the lesser number:

$$\text{Then let}\left\{\begin{array}{l} a+e=s=28 \\ a-e=d=14 \\ ae=p=147 \\ \frac{a}{e}=q=3 \\ aa+ee=z=490 \\ aa-ee=x=392 \end{array}\right\}\text{their}\left\{\begin{array}{l}\text{Sum,} \\ \text{Difference,} \\ \text{Product,} \\ \text{Quotient,} \\ \text{Sum of their squares,} \\ \text{Differ. of their squares.}\end{array}\right.$$

Any two of these six letters (s, d, p, q, x, z,) being given, to find the rest; which admits of 15 variations or questions, and are those which Mr. Ward has so excellently answered in his MATHEMATICIAN'S GUIDE.

Question 1. Suppose s and d were given, and it were required by them to find a, e, p, q, z, and x?

(2) Let s and p be given, to find the rest?
(3) Suppose s and q were given, to find the rest?
(4) Suppose s and z were given, to find the rest?
(5) Let s and x be given, to find the rest?
(6) Suppose d and p are given, to find the rest?
(7) Let d and q be given, to find the rest?
(8) Suppose d and z are given, to find the rest?
(9) Suppose d and x were given, to find the rest?
(10) Let p and q be given, to find the rest?
(11) Suppose p and z be given, to find the rest?
(12) Let p and x be given, to find the rest?
(13) Suppose q and z were given, to find the rest?
(14) Let q and x be given, to find the rest?
(15) Suppose z and x were given, query the rest?

These fifteen questions being pursued throughout, to find the value of all the unknown quantities will afford a great variety, and will be found to be of great service to the pupil, in the solution of most questions.

Note.—I would advise the learner to use the same numbers, for the respective value of each quantity throughout all the questions, as they will be more satisfactory in proving the work than various numbers will be; not but any number may be taken at pleasure, provided the number represented by a be greater than that by e, &c.

The pupil may make a numerical calculation in each question.

PROBLEMS.

(1) What two numbers are those, whose difference is 20, and whose sum is 70?

(2) What two numbers are those, whose difference is 14, and the quotient of the greater divided by the less is 3?

(3) What number is that, whose third part, added to its fourth, will be 21?

(4) What number is that, whose third part exceeds its fourth by 4?

(5) What number is that, whose third part, less 25, is equal to its fourth, less 4?

(6) What numbers are those, whose difference is 8, and the difference of their squares is 208?

(7) What two numbers are those, whose sum is 60, and the greater is to the lesser, as 9 is to 3?

(8) Find two numbers, the product whereof is 108, and the triple of the greater divided by the lesser is 4.

(9) Find two numbers, to whose sum if you add 8, the whole will be double the greater; and if you subtract 4,5 from their difference, the remainder will be half the least.

(10) Find three numbers, so that the first and half the remainder, the second and one-third of the remainder, and the third and one-fourth the remainder, may always make 34.

(11) Divide 100 twice in two parts, so that the major part of the first division may be three times the minor part of the second division, and the major part of the second may be double the minor part of the first.

(12) Three persons, A. B. and C. make a joint contribution, which in the whole amounts to 76*l.* Of this A. contributes a certain sum unknown; B. as much as A. and 10*l.* more; and C. as much as both A. and B. together. I demand their several contributions.

(13) There are 480 men to be placed in an oblong, whose length and breadth together make 52. How many in each front?

(14) Sold a quantity of tobacco for 19*s.* part of which at 1*s.* per pound, and the rest at 15*d.* Now the first part was to the latter as $\frac{3}{4}$ to $\frac{2}{3}$. How much was sold of each sort?

(15) After paying away $\frac{1}{4}$ and $\frac{1}{5}$ of my money, I found 66 guineas left in my bag. What was in it at first?

(16) What two numbers are those, whose sum, multiplied by the greater, produces 77; and whose difference, multiplied by the lesser, gives 12?

(17) What number, from whose double 10 being subtracted, the square of the remainder, less 1, will be equal to 7 times the said number?

(18) A father at his death left, by his will, 1000*l.* to be divided between his son and daughter, in such a manner that $\frac{1}{5}$ part of his share should exceed $\frac{1}{4}$ part of hers by 10*l.* How must the 1000*l.* be divided?

(19) The continual product of four numbers in arithmetical progression is 945, and the common difference 2. What are those numbers?

(20) If to my months you should add half this sum,
And one-eighth more, and then should subtract one,
The residue would such a number be
As twenty-one, being squared, assuredly.

(21) Three numbers in geometrical progression are required, so that the difference of the first and second may be 6, and of the second and third 15.

(22) To find a number, from the cube of which if you subtract 19, and multiply the remainder by that cube, the product shall be 216.

(23) Sixty thousand brave soldiers in battle there were
Plac'd on a vast plain, and in form a long square:
Now on how many acres of ground did they stand,
At two yards three quarters between man and man?
And how many in rank and file will there be,
When their breadth to their length is as two are to three?

(24) A general, disposing of his army into a square battle, found he had 284 soldiers over and above; but increasing each side with one man, he wanted 25 to fill up the square. Query, the number of soldiers?

(25) Old John, who had in credit liv'd,
Though now reduc'd, a sum receiv'd.
This lucky hit's no sooner found,
Than clam'rous duns came swarming round.
To th' landlord—baker—many more,
John paid, in all, pounds ninety-four.
Half what remain'd a friend he lent;—
On Joan and self one-fifth he spent;

And when of all these sums bereft,
One tenth o' th' sum receiv'd had left.
Now show your skill, ye learned youths,
And by your work the sum produce.

(26) In a rectangle A B C D, is given the difference between the length, A B, and the diagonal, B D, that is, D E=2; likewise the difference between the breadth, A D, and the diagonal, B D, that is, F D=9; required the sides of the the rectangle A B, A D.

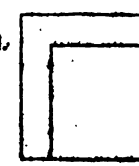
(27) In a triangle, A B C, the several sides are given, viz. A B=15, A C=14, B C=13, and the perpendiculars C D being drawn. Required the segments of the base B D, D A.

(28) Suppose the plate of a looking-glass is 18 inches by 12, and is to be framed with a frame of equal width, and whose area is to be equal to that of the glass; the width of the frame is required.

(29) One morning in May I went to survey,
As soon as bright Sol I espy'd;
I measured round a four-corner'd ground,
The margin's* the length of each side:
The angle at B, together with D,
A hundred and fifty degrees;
The meadow's content is all that I want:
Assist me, kind youths, if you please.

* A B=15.60
B C=13.20
C D=10.00
D E=26.00

(30) A gentleman a garden had,
Five score feet long, and four score broad,
A walk of equal breadth, half round,
He made, that took up half the ground:
The figure in the margin see.
How wide the walk, I pray tell me?



(31) Ye skilful artists, be so kind
The fences of a field to find;
Triangular the form of ground;
Its area twice twelve chains was found;
One at the base, observe, I knew,
Just thirty-six and fifty-two;
And cub'd when every side shall be,
The sum you in the margin see.*

* 1728 chains.

For more examples, see Sect. 30 and 31.

A few diverting QUESTIONS.

(1) A Cheshire cheese, when in one scale, weighed 76 lb.; but on being changed into the other scale, it weighed only 56 lb. Query, the true weight? Ans. 65,1965 lb.

The general rule for questions of this sort is, to multiply the two weights together, and extract the square root of the product, which root will be the true weight.

(2) A stone, weighing 40 lb. is by accident broken into four pieces, by which may be weighed any number of pounds, from 1 lb. to 40 lb. Query, the weight of each piece?

A General RULE for the Solution of QUESTIONS of this Nature.

To double the first or least weight, which always contains one pound, add 1, and it gives the second weight.

Again, to double the sum of these two weights, add 1, it produces the third weight: and again, to double the sum of these three weights, add 1, and we shall have the fourth weight.

(3) A certain company being at a public house, their reckoning came to 6*s.* 4*d.* The number of persons in company were equal to the farthings each spent. Query, the number in company, and what each spent?

The general rule to answer questions of this sort is, to reduce the given sum to its lowest name mentioned, and of that extract the square root, which will be the answer.

A pack of cards being laid into any number of heaps, so that the spots on the bottom card of each heap, added to the number of cards laid thereon, may make 12, by giving the number of heaps, and of the cards left out, to find the number of spots on all the bottom cards.

QUESTIONS of this nature are solved by the following Theorem:

Suppose n=the number of heaps,
r=the number of cards left,

w = the whole pack

x = the number of spots required.

Then the Theorem will be $13 \times n - w + r = x$, the number of spots required.

(4) Suppose a pack of cards be dealt into seven heaps, and then there are 12 cards left out. Query, the number of spots on all the bottom cards?

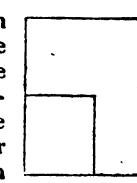
(5) What dimensions must I give to a joiner to make a cubical box that will hold 2000 oranges of $2\frac{1}{2}$ inches diameter each, supposing the oranges globular, keeping that form, and laid in rows exactly at the top of each other? Ans. 31,498 inches inside.

(6) A master-joiner gives to one of his men a plank that is 10 feet by 2, with orders to make of it a square table, equal in area to the said plank, but not to exceed five segments: the poor man being ignorant of lines (and not willing his master should know it), would be greatly obliged to any one who will draw the plan how the said plank must be cut and applied together.

(7) A person being asked what hour of the day it was, answered, it is between 5 and 6, and both the minute hand and hour hand are together. Required the hour of the day?

(8) A lady has a dressing-table, each side of which is 27 inches: but she is desirous to know how each side of the same may = 36 inches, by having 4 feet of plank, superficial measure, joined to the same. The plan in what manner the plank must be cut and applied to the table is required.

(9) A gentleman purchases a piece of land in form of a parallelogram, and encloses one fourth part (as per figure) to build a house and other conveniences upon: now he desires the remainder of this land may be divided into four parts, equal and similar to each other, to be appropriated to such uses as he shall hereafter think proper. The plan is desired.



(10) Required to divide a given trapezium, of two equal parts, geometrically, by the shortest line possible.

(11) A person, for a considerable wager, is to travel from a town A. to another B. but he is to call at a place which is somewhere on the line $a\,b$. Now his time

limited being very short, he is desirous to know (by geometry) the situation of the place on the line *a b*, the distance from thence to A. and B. being the shortest possible.

(12) All the different ways possible in which a gentleman can place his servants, combining them by 1, 2, 3, &c. at a time, are 960799. What number of servants does he keep?

APPENDIX.

APPENDIX;

CONTAINING

Various Forms of Acquittances, Promissory Notes, Bills of Exchange, Letters of Advice, Letters of Credit, Bills of Parcels, and Bills on Book Debts; all of which are adapted to such circumstances as occur in real Business, to enter a Pupil in the manner and Method of Commerce, and to make him ready at Computation.

A general Receipt.

RECEIVED, August 9th, 1806, of Mr. Anthony Buskin, the sum of fifty pounds, in full for one quarter's rent, due at Midsummer last, and of all demands,

£ 50 00 00 By me, HENRY HEDGES.

A Receipt or Acquittance for Rent paid.

RECEIVED, this 5th day of January, 1806, of Mr. Thomas Field, the sum of thirty-two pounds sixteen shillings, in money, which, with eight pounds four shillings more, disbursed by the said Thomas Field, for taxes and reparations of the messuage and tenements he now occupies, situate in Chiswell-street, London, makes in the whole the sum of forty pounds, and is in full of half a year's rent

due to me out of the said premises at Christmas-day last.—I say, received by me,

£ 40 00 00 THOMAS ROSE.

An Acquittance for Debt received of a Third Hand.

RECEIVED, this 10th day of July, 1806, of Mr. Joseph Stanley, by the hands of Mr. Clark Stanley, the sum of seven pounds, in full for certain goods bought by the said Joseph Stanley of me.—I say, received in full of all demands, by me,

£ 7 00 00 FRANCIS CRUMP.

An Acquittance for Money received in Part of a Debt due on Bond.

RECEIVED, this 16th of June, 1806, of Mr. Thomas Brown, the sum of ten pounds ten shillings and sixpence, in part of payment of a greater sum, due to me on bond from the said Thomas Brown.—I say, received by me,

£ 10 10 6 THOMAS HEDGES.

An Acquittance for Money received by a third Person for the Use of another.

RECEIVED, this 18th day of March, 1806, of Mr. Edward Hedges, the sum of twenty pounds, in full for work done by Charles Day, for the said Mr. Edward Hedges.—I say, received, by the order and for the use of the said Charles Day, by

£ 20 00 00 THOMAS WOOTON.

A Receipt for Interest due on Bond.

RECEIVED, this 13th day of January, 1806, of Mr. Abraham Brooks, the sum of five pounds, in full for one year's interest of 100*l.* due to me at Christmas last, on bond from the said Abraham Brooks.—I say, received by me,

£ 5 0 0 DAVID BURN.

Note.—Besides these receipts to be taken on payment of money due on bonds, it is proper to have each payment mentioned on the back of the obligation.

An Acquittance for a Legacy.

RECEIVED, this 29th day of July, 1806, of John Roberts, executor of the last will and testament of Samuel Green, late of Westham, in the county of Essex, deceased, the sum of one hundred pounds, in full of a legacy bequeathed to me, in and by the last will and testament of the said Samuel Green. I say, received in full of all demands, by me,

£ 100 0 0 JOHN JARRET.

An Acquittance to an Administrator on Payment of a Debt due from the Intestate.

RECEIVED, this 30th day of July, 1806, of Mr. William Jarret, administrator of the goods and chattels, rights and credits, of John Noon, late of Bristol, in the county of Somerset, deceased, the sum of three hundred pounds, in full of a debt, owing to me by the said John Noon in his life-time, for household goods by me sold him. I say, received in full of all demands, by me,

£ 300 00 00 GEORGE CHINNER.

A Receipt proper to be taken upon a Person's giving a Promissory Note for a Book Debt.

RECEIVED, this 20th day of April, 1806, of Mr. William Straight, a promissory note for the sum of sixty pounds, payable to me, or order, three months after date; which sum, when paid, is in full of all demands.—I say, received by me,

£ 60 00 00 THOMAS ROSS.

An Acquittance for the Purchase-Money on executing a Conveyance, to be indorsed on the Back of a Deed.

RECEIVED, the day and year within written, of the within-named Thomas Bisson, the sum of seventy pounds, being the full consideration-money within-mentioned to be paid to me.—I say, received by me,

£ 70 00 00 CHARLES BARRON.

Witness to the payment of the money.

A Receipt for Writings intrusted in a Person's Hands.

RECEIVED, this 25th day of November, 1806, of John Stag, of Eye, in the county of Suffolk, four several deeds or conveyances: one of them purporting to be a lease of a messuage, situated in Narrow-street, Limehouse, and made between John Lee, of Wapping, and William Burt, of Westminster; another to be an assignment of the said lease, and made between John Vyse, of New-street, London, and Felix Morgan, of Bangor, in the county of Caernarvon; and the other to be a lease and release, and made between John Jee, of Barking, in the county of Essex, and John Beven, of Newhaven, in the county of Sussex; for which several deeds or writings, I hereby promise to be accountable, and to re-deliver the same to the said John Stag on demand.

Witness my hand,

HENRY WRIGHT.

The Form of Promissory Notes, or common Notes for Money.

The Form of one payable on Demand.

I promise to pay to Anthony Wilson, or order, on demand, the sum of twenty pounds, for value received.—Witness my hand, this first of January, 1806.

£20 00 00 HENRY WILSON.

The Form of one payable at a certain time.

London, January 4, 1806.

THREE months after date, I promise to pay to Mr. Aaron Brooks, or order, the sum of ten pounds, for value received by me,

CHARLES VYSE.

£10 00 00

Form of one payable at sundry Times.

I promise to pay Sir Aaron Day, or order, the sum of thirty-six pounds, in the manner following: ten pounds, part thereof, three months after date; sixteen pounds more the 20th of May next; and the remaining ten pounds the 27th of July next following, for value received.—Witness my hand, at London, the 2d of January, 1806.

JOHN COWCHER.

£36 00 00

Form of one for Goods received.

I promise to return John Whyley, Esq. or order, on demand, one casket of jewels sealed, one hundred ounces of gold plate, three hundred and fifty of silver ditto, one hundred carats of oriental pearls, and a five hundred pound bank note, received of the said John Whyley, Esq. for self and company, March 4th, 1806, by me,

JAMES COWCHER.

Form of Inland Bills of Exchange.

Form of one payable at Sight.

£ 100

Bristol, January 14, 1806.

AT sight pay Mr. John Barwick, or order, the sum of one hundred pounds, the value received of Mr. James Barwick, and place it to account, as per advice from

WILLIAM BARWICK.

To Mr. Charles Davis, at the Horse Guards, Whitehall, London.

Form of one payable after Sight.

£ 64 17 6

London, June 12, 1806.

AT ten days sight pay Mr. William Straight, or order, the sum of sixty-four pounds seventeen shillings and sixpence, the value received from Thomas Johnson, Esq. and place it to account, as per advice from

GEORGE DEAR.

To Mr. Thomas Lawr, Clothier, Salisbury.

Form of one payable after Date.

£ 70

Edinburgh, July 2, 1806.

THREE months after date, pay Sir John Walker, or order, the sum of seventy pounds sterling, value in ourselves, and place it, without more advice, to the account of

THOMAS and JAMES BURN.

To Sir Thomas Bisson, Lombard-street, London.

£ 40

London, Nov. 24, 1806.

Messrs. Fox and Payn.

PAY Thomas Stanley, or bearer, forty pounds on account.

THOMAS PETTIT.

Another.

SIR, York, June 4, 1806.

PAY Mr. Thomas Ross, or bearer, one hundred pounds, on demand, and place it to my account.

THOMAS CLIFFORD.

To Mr. John Hawkins, silversmith, Whitechapel.

Form of Foreign Bills of Exchange.

For Crowns 600 *at Usance.*

London, July 12, 1806.

London on Paris. First Bill. AT usance pay this first of exchange to Henry Gibbons, or order, six hundred crowns, for the value here received of Samuel Drummond, and place it to account, as per advice from

JOHN PLAW.

To Mr. George Pain, merchant, at Paris.

For Crowns 600 *at Usance.*

London, July 12, 1806.

Second Bill. AT usance pay this my second of exchange, my first not paid, to Joseph Lindman, or order, six hundred crowns, for the value received of Sir George Wayer, and place it to account, as per advice.

THOMAS SWIFT.

To Mr. Robert Clifford, merchant, at Paris.

£ 602 15*s.* 3¼*d.* Sterl. at 34¼*d.* per £ Sterl. usance.

London, Sept. 6, 1806.

London on Rotterdam. First Bill.

AT usance pay this first of exchange to James Vyse, or order, six hundred and two pounds fifteen shillings and three pence one farthing Sterl. at thirty-four pence one farthing Flem. per pound Sterl. value of Samuel Turner, Esq. and place it as per advice from

Your most humble servant,

THOMAS WOOLDRIDGE.

To Mr. Samuel Cook, merchant, at Rotterdam.

Form of a Factor's Remittance to his Employer.

For Crowns 700, at 54*d.* Sterl.

Genoa, Sept. 6, 1806.

AT twenty days sight pay this my first of exchange to Thomas Godfrey, or order, seven hundred crowns, exchange at fifty-four pence per crown, value received of the Lords of the Regency, and place it to the account of Mr. William Stanes, of Lyons, as per advice from him.

GEORGE WARD.

To Mr. Francis Baker, banker, Lombard-street, London.

Mr. William Stanes's Letter, advising his Concurrence to the afore-mentioned Draught.

SIR, Lyons, Sept. 30, 1806.

Mr. George Ward, of Genoa, has this day desired me to furnish him with seven hundred crowns, payable to the resi-

dent of the States at London. I have therefore ordered him to draw for the said sum on you, which please to honour as usual, and put to the account of

Your friend and servant,

WILLIAM STANES.

To Mr. Francis Baker, banker,
Lombard-street, London.

Form of a Draught on the Employer for Value of Goods shipped him per Factor.

Milrees 400 at usance.

St. Andrews, June 25, 1806.

North Britain on Lisbon. AT usance pay this my first of exchange to Mr. Edward Ross, or order, four hundred milrees, the value here shipped for your use upon the Santa Maria, of Naples, and consigned as per advice from your very humble servant,

MICHAEL GODFREY.

To Signor Santilena,
at Lisbon.

Form of an Employer's Letter with Remittance to his Factor, in a Bill of the said Factor's Correspondent.

Mr. Thomas Dale,

ACCORDING to your desire I have remitted you four hundred crowns for my account, in your correspondent Baker's bill inclosed, payable by and to yourself, for which please to give me credit. I recommend the contents of my last and the 4th current to you, and rest your friend and humble servant,

EDWARD BAKER.

Paris, Sept. 14, 1806.

For crowns 400, at 34*d.* per crown.

Paris, Sept. 14, 1806.

The bill. AT double usance pay this my only bill of exchange to myself, the sum of four hundred crowns, exchange at thirty-four pence sterling per crown, the value received of Mons. Edward Bakerman, and place it as per advice to the account of

Your humble servant,

HENRY VYSE.

To Thomas Day, merchant, at Bristol.

Form of the Correspondent's Letter of Advice.

Mr. Thomas Day,

SIR, Paris, Sept. 14, 1806.

BY this post I have drawn on you for four hundred crowns, at 34*d.* payable to yourself, value of Mons. Edw. Bakerman, which, with my other bills depending, please to honour, and the timely remittance shall be punctually made you by,

Sir,

Your very humble servant,

NICHOLAS BROWN.

To Mr. Timothy Bevan, merchant, Bristol.

Form of Letters of Credit.

SIR, London, June 12, 1806.

PLEASE to furnish the bearer hereof, Mr. Thomas Cavendish, with the sum of one hundred pounds, as he shall require the same, and place it to my account: for which this letter of credit, and his receipt, shall be your sufficient

voucher and warrant; giving upon payment a line or two of advice to

Your sincere friend,

FRANCIS ROSS.

To Mr. John Day, merchant, at Hull.

The Receipt.

RECEIVED, July 6, 1806, of Mr. Robert Berry, the sum of one hundred pounds, by virtue of Mr. Francis Ross's letter of credit, of June the 12th last, for the said sum, per

MICHAEL NICHOLS.

£ 100 0 0

SIR, Dublin, May 7, 1806.

The bearer, Mr. Thomas Vyse, will have occasion for sixty pounds, which sum I desire you to furnish him, and take his bill for the said sum, or any part thereof, on the Honourable William Webb, Esq.

I am, Sir,

Your humble servant,

RICHARD KIRK.

To Henry Hedges, Esq. Fleet-street, London.

Exch. £70.

SIR, June 4, 1806.

The Bill. AT twelve days' sight pay this my first of exchange to Henry Hopkins, Esq. or order, the sum of seventy pounds, the value received of ditto for your use, as per advice from, Sir,

Your humble servant,

THOMAS VYSE.

To the Hon. William Bay, Esq. Dublin.

Exch. £70.

SIR, June 4, 1806.

AT twelve days' sight pay this my second of exchange, my first not paid, to Richard Kirk, Esq. or order, the sum of seventy pounds, the value received for your use, of Henry Hopkins, Esq. as by advice from

Your humble servant,

THOMAS WILKS.

To the Hon. William Bay, Esq. Dublin.

Form of a General Letter of Credit, to furnish a Person according to his occasion.

SIR, Paris, May 12, 1806.

THE bearer, Mr. Aaron Babel, one of his Britannic Majesty's messengers, being ordered to Constantinople, will have occasion for money to defray his charges, &c. Please to furnish him with the sums he shall require at the said place, taking his receipts; and your draught, for the value, shall receive due honour from

Your humble servant,

MICHAEL WELLS.

Mons. Carter Day, Banquier, à Vienne.

For 600 Florins, at 3 *liv.* 4*s.*

SIR, Vienna, June 2, 1806.

AT three days' sight pay this my only bill of exchange to Edward Fay, or order, the sum of 600 florins, exchange at three livres four sols per florin, the value paid at Constantinople to Aaron Babel, pursuant to your letter of credit of the 12th of May last, and as by advice from the said Aaron Babel.

RICHARD STANES.

A Mons. Henry Hedges, Banquier, à Paris.

BILLS OF PARCELS AND BOOK DEBTS.

(1) Mr. A. B.

Bought of THOMAS GROCER, Jan. 2, 1806.

		s.	d.		£. s. d.
4 lb. of green tea,	at	17	6	per lb.	
12 lb. of bohea,		7	10¾	——	
5 Hundred of tobacco,		66	0	per c.	
10 Barrels of raisins,		46	4	per b.	
9 lb. of pepper,		2	9	per lb.	
7 lb. of brimstone,		1	8	——	
8 lb. of coffee,		12	9	——	
77 oz. of mace,		1	6¼	per oz.	
				£.	

(2) Mrs. T.

Bought of THOMAS SILK-MERCER, Feb. 7, 1806.

		s.	d.		£. s. d.
20 Yards of brocaded satin,	at	17	9	per yard	
36 Yards of mohair,		10	8	——	
86 Yards of green silk damask,		19	11½	——	
100 Yards of paduasoy,		11	10	——	
9 Yards of double taffety,		3	9	——	
12 Yards of ditto,		5	6	——	
6 Yards of ditto,		9	11½	——	
42 Yards of ditto,		10	6	——	
				£.	

(3) The Hon. Lady B.

Bought of J. LINEN-DRAPER, March 6, 1806.

		s.	d.		£. s. d.
100 Ells of dowlas	at	0	11¾	per yard	
96 Ells of holland,		5	7½	——	
19½ Ells of diaper		1	7	——	
27½ Yards of damask,		4	10¼	——	
12 Yards of muslin,		11	0	——	
24½ Ells of cambric,		5	6	——	
				£.	

(4) Madam B.

Bought of B. Milliner, April 4, 1806.

		s.	d.		£. s. d.
26½ Yards of blue riband,	at	2	6	per yard	
12 Fans, French paper mount,		4	9	each	
6 Pair of Roman gloves,	—	9	6	per pair	
4 Dozen of Irish lamb ditto,		1	10½	——	
6 Sarsenet hoods, white,	—	5	11	each	
A piece of Mechlin lace, 12¼ y.		17	6	per yard	
				£.	

(5) Mrs. F. Pindust,

Bought of Thomas Hosier, May 12, 1806.

		s.	d.		£. s. d.
28 Pair of thread stockings,	at	4	6	per pair	
1½ Dozen of worsted mixed,	—	3	10	——	
18 Pair of strawberry hose,	—	4	3	——	
16 Pair of silk gloves,	—	5	11½	——	
74 Pair of Norwich hose,	—	2	6	——	
17 Pair of men's yarn,	—	3	0	——	
11½ Dozen ditto, silk,	—	16	6	——	
				£.	

(6) Mrs. Ann Scott,

Bought of John Fruiterer, June 7, 1806.

		s.	d.		£. s. d.
6 Dozen of lemons,	—	at 2	10	per doz.	
4½ Hundred of Lisbon ditto,	—	6	6	per hund.	
10 Ropes of onions,	—	1	9	each	
17 Doz. Seville and China oranges,		4	6	per doz.	
A bushel of chesnuts,	—	10	6		
6 Dozen of pomegranates		4	10	per doz.	
				£.	

(7) Mr. C. Toms,

Bought of James Draper, July 4, 1806.

		s.	d.		£. s. d.
12⅞ Yards of broad cloth,	at	18	6	per Yard	
17 Yards of fine Spanish black,		17	4	———	
14⅛ Yards of fine grey cloth,		16	10	———	
6½ Yards of second drab, —		12	8	———	
27¾ Yards of shalloon, — —		1	9	———	
19 Yards of serge, — —		4	10½	———	
				£.	

(8) Mr. W. L.

Bought of Andrew Oilman, Aug. 7, 1806.

		s.	d.		£. s. d.
12 lb. of anchovies, —	at	1	6	per lb.	
6½ of capers, — —		0	11½	———	
26 lb. of salt, — —		0	9¾	———	
4 gallons of pickled mushrooms,		3	6	per gal.	
3 gallons of Lucca oil, —		10	5	———	
12 lb. of saltpetre, —		1	4¾	per lb.	
				£.	

(9) John Jones, Esq.

Bought of Thos. Cheesemonger, Sept. 16, 1806.

	C.	qr.	lb.		s.	d.		£. s. d.
6 Cheshire cheeses, wt.	3	2	14	at	44	6	per C.	
3 Gloucester ditto,	2	3	21		36	4	———	
8 Flitches of bacon,	50 stone				2	11½	per st.	
10 Firkins of butter,	—				32	6	each	
24 Stilton cheeses	6	2	9		37	10	per C.	
3 Weys of Suffolk butter					3 12	6	each	
							£.	

(10) Sir Matthew Lamb,

Bought of Nath. Winemerchant, Oct. 4, 1806.

			s.	d.		£. s. d.
12½	Dozen of claret wine,	at	37	6	per doz.	
4	Gallons of Canary,	—	9	6	per gal.	
2	Hhds. of old mountain,		6	4	——	
26	Dozen of Madeira,	—	30	6	per doz.	
12	Dozen of tent,	—	59	10	——	
4½	Dozen of red port	—	19	10	——	
					£.	

(11) Mrs. B.

1806. Bought of J. Butcher.

			lb.	d.		£. s. d.
Jan. 2.	A fore quar. of lamb, wt.		10¾	at 4¾	per lb.	
29	Ditto of pork,	—	14	5	——	
Feb. 4.	A buttock of beef,		49	4½	——	
10.	A fillet of veal,	—	11½	5¼	——	
27.	A sirloin of beef,	—	26	4½	——	
Mar. 6.	Beef steaks,	—	10	5¾	——	
14.	A saddle of mutton		17¼	4¾	——	
					£.	

(12) John Thomas,

Bought of J. Cornchandler, Oct. 4, 1806.

			s.	d.		£. s. d.
4	Quarters of barley,	at	16	9	per quar.	
12	Bushels of wheat,	—	5	9	per bush.	
7	Ditto of oats,	—	11	9	per quar.	
20	Bushels of beans,	—	4	6	per bush.	
18	lb. of hops,	—	1	9	per lb.	
4	Loads of hay,	—	50	0	per load	
					£.	

(13) Mr. Jones, Dr. To J. Coalmerchant.

1806.			s.	d.		£. s. d.
Feb. 17.	4 Sacks of coals,	at	39	6	per ch.	
Mar. 4.	5½ Chaldrons of ditto,		30	0	———	
16.	8 Sacks of ditto,	—	34	6	———	
27.	9 Sacks of ditto,	—	42	0	———	
April 6.	7 Sacks of ditto,	—	38	9	———	
20.	3 Sacks of ditto,	—	45	0	———	
					£.	

(14) Thomas Johnson,

Bought of John Grocer, Nov. 27, 1806.

	C. q. lb.		s.	d.		£. s. d.
12 Hhds. of tobacco, wt.	6 1 24	at	53	6	per c.	
9 Barrels of raisins,	2 3 20		39	8	———	
12 Ditto of rice,	3 1 27		26	11	———	
8 Bags of pepper,	1 3 14		79	6	———	
Brimstone,	3 2 24		27	6	———	
4 Hhds. of sugar,	4 1 17		33	9½	———	
					£.	

(15) Mr. John Man,

Bought of John Jee, Dec. 27, 1806.

6 Casks of Barbadoes sugar, at 2 months' credit, viz.

			C. q. lb.	
No. 1.	———	wt.	10 0 24	Tare 3 *qrs.* 10 *lb.* each.
2.	———	—	9 2 17	
3.	———	—	8 3 20	
4.	———	—	11 2 17	
5.	———	—	10 1 1	
6.	———	—	9 3 19	
		Gross,		
		Tare,		
		Net,		at 53*s.* 6*d.* per cwt.

(16) Mr. John Ross,

Bought of John Mackinder, Jan. 30, 1806.

		C.	q.	lb.		s.	d.		£. s. d.
3 Hhds. of sugar,	Gross	10	3	10					
	Tare	0	2	10	at	74	0	p. c.	
2 Bar. of Scotch snuff,	Gr.	8	2	7					
	Ta.	0	2	7	at	1	6	p. lb.	
2 Ditto of pimento,	Gr.	7	3	7					
	Ta.	0	2	2	at	1	10	——	
3 Ditto of figs,	Gr.	8	2	11					
	Ta.	0	1	27	at	30	0	p. c.	
4 Hhds. of treacle,	Gr.	11	2	19					
	Ta.	0	2	27	at	14	10	——	
3 Ditto of prunes,	Gr.	10	1	11					
	Ta.	0	2	25	at	17	6	——	
								£.	

(17) Mr. Thomas Bail,

Bought of John Silversmith, June 3, 1806.

		oz.	dwt.	gr.		s.	d.		£. s. d.
A silver tankard,	wt.	14	2	3	at	6	9	p. oz.	
—— punch ladle,		0	19	20	at	7	2	——	
—— punch bowl,		24	1	7	at	7	6	——	
A pair of sauce-boats,		40	11	10	at	7	10	——	
6 Cruet tops,		4	9	3	at	6	4	——	
2 Sauce-pans,		62	19	4	at	7	2	——	
1 Doz. of dishes, each		26	14	6	at	6	2	——	
Ditto of plates,		14	10	9	at	6	8½	——	
								£.	

THE END.

JUL 12 1920

Printed by S. Hamilton, Weybridge, Surrey.